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patch = Stück, Flicken ≙ Polygon...?
blend = mischen, verbinden

Surface Generation from an Irregular Network of Parametric Curves

Shigeru Kuriyama
Tokyo Research Laboratory, IBM Japan, Ltd.
1623-14 Shimotsuruma Yamato-shi Kanagawa 242, Japan

Abstract

This paper proposes a method of generating surfaces from a network of curves that have arbitrary parametric forms, and that intersect in an arbitrary topology.

The surfaces generated from the network are represented by multisided patches defined on a multivariate coordinate system. An m-sided patch is generated by blending m sub-surfaces with a transfinite interpolant, and each sub-surface is generated by blending two sweep surfaces that are defined by a pair of curves intersecting with each other in the network. An advantage of the final surfaces is that they have everywhere the same order of continuity as the curves.

This method is flexible in its representation of the curve expressions and the connective topology of a network. It can implement a surface model in user-friendly and designer-oriented CAD interfaces that handle direct input of 3D curves.

Key Words: Computer-aided geometric design, Multisided patches, Sweeping, Blending, Geometric continuity, Curve network.

1 Introduction

New technologies in computational graphics, such as photo-realistic rendering, enhance the visual effect of presentations in industrial design and commercial production. The input of shapes, however, is still a time-consuming process, and needs the support of skilled CAD operators.

A new paradigm is required for current 3D-CAD systems used to design the shapes of free-form surfaces, because the systems are inadequate for quick input of shapes in the initial stage of conceptual design. 3D-CAD systems have come to use the metaphor of sketching that was used by 2D-drawing systems in order to realize a designer-oriented environment of shape input. Sketching systems aim at a user-friendly interface that is easy to use, intuitive, and good at handling complicated shapes.

In current surface modellers, free-form surfaces have a topological constraint on their control points or profile curves; they must be arranged in the topology of a regular two-dimensional mesh. This constraint is derived from the formation rule of tensor product surfaces. 3D-sketching systems based on current surface modellers therefore generate tensor product surfaces such as loft or sweep surfaces from a set of hand-drawn curves arranged in the topology of a regular mesh. This topological constraint, however, limits the expressional flexibility of designers.

On the other hand, advanced three-dimensional input devices have been proposed for CAD systems whose interface handles the input of free-form curves. For example, an MIT group (Sachs, Roberts and Stoops, 1991) has developed a CAD interface that allows users to design shapes by entering information of free-form curves directly in three dimensions, using a pair of hand-held sensors. The interface manages the input of a curve network that is free from the topological constraint of a regular mesh. We call such a network *irregular*. The network, however, represents only a wireframe model and lacks a surface model. This limitation of the representative model makes it impossible to conduct engineering evaluations and simulations such as interrogation or rendering of surfaces, or data generations for Numerical Control machine or Finite Element Method. The above example highlights the need for a method of skinning an irregular network of curves.

Multisided patches have the potential to generate smooth surfaces from an irregular network of curves, because they can have an arbitrary number (more than two) of sides corresponding to their boundary curves. The existing methods of generating multisided patches have the following common drawbacks:

- Each boundary curve of a patch must contain only one segment; it is always defined by one expression.
- It is impossible or very complicated to generate curvature-continuous surfaces.

it is designed by sketching. Moreover, designers using surface modellers in industrial design often impose strict conditions on the continuity of surfaces so that shapes will have a smooth appearance. They claim that G^2 continuity (that is, uniqueness of surface normals and of either principal curvatures or principal directions) is necessary in order to confirm the shape from the continuous reflection curves on the surfaces. However, the existing methods are inadequate to satisfy these conditions.

For these reasons, we have improved the mathematical model of multisided patches to match data incidents to corresponding edges, and to ensure geometric continuity of arbitrary degrees.

In this paper, we propose a method of skinning a network with multisided patches. These are generated by sweeping and blending the curves corresponding to the boundaries of the patches. This method can implement a surface model in CAD interfaces that handle only a wireframe model by allowing direct sketching of curves in three-dimensional space.

In Section 2, we explain the existing methods of generating multisided patches and sweep surfaces. In Section 3, we propose a method of generating a multisided patch from curves surrounding the patch. In Section 4, the method proposed in Section 3 is modified for singular conditions on the topology of the network: patches that have multiple and T-connected intersections, and open-sided and two-sided patches are considered. In Section 5, we give examples of curve networks and surfaces generated from them by our method, and in Section 6, we offer some conclusions and discuss future work. The Appendix includes a proof of the geometric continuity of the patches defined in Section 3.

2 Previous Work

Methods of generating multisided patches have become important as a result of the need for a mathematical model that can handle complicated shapes. These methods eliminate the drawback of tensor product surfaces; namely, a constraint on the arrangement of control points or profile curves.

Catmull and Clark (1978), Doo and Sabin (1978), and Nasri (1987) proposed the recursive subdivision method, to remove the restriction on the topology of surfaces. This method, however, does not have closed-form parametrization. Hosaka and Kimura (1984), and Loop and DeRose (1989) introduced multisided patches, which are regarded as a generalization of Bézier patches for multivariate barycentric coordinates. Loop and DeRose (1990) presented a method of constructing multisided B-spline surfaces with multisided patches, called S-patches, by using Sabin nets (Sabin, 1983). However, it is hard to calculate the control points of these patches in such a way as to generate G^2 continuous surfaces.

Varady (1991) proposed a method of overlapping patches, introducing local parametrization for individual vertex patches, and Charrot and Gregory (1984) introduced multisided patches by using local parametrization of a multivariate coordinate system and a convex combination of blending functions. Nielson (1987), and Hagen and Pottmann (1989) also proposed triangular patches defined on barycentric coordinates through the use of blending functions. Their methods are similar to Charrot and Gregory's, and are extendible to multisided patches. These methods can generate G^2 surfaces by increasing the degree of constituent equations, and they have no constraints on the representation of sub-surfaces to be blended.

The above-mentioned methods of generating topologically free surfaces are still used for patches that match data incidents only to corresponding vertices. That is to say, the surfaces are defined by geometrical values assigned to corresponding corners of the patches.

On the other hand, methods of generating sweep surfaces are well known and are implemented on most surface modellers, because they allow the design of shapes to be curve-based rather than vertex-based.

Woodward (1988) proposed techniques for skinning surfaces by using interpolation based on B-splines, and Coquillart (1987) described a method based on non-uniform rational B-splines by adding a profile curve to scale the inbetween cross sections. Choi and Lee (1990) classified sweeping rules as parallel, rotational, spined, and synchronized sweeps; these sweeps are generalized by combining coordinate transformation and blending. Klok (1986) proposed a method of sweeping along a 3D trajectory by using rotation minimizing sweep that is a modification of Frenet frame sweep, and Tai, Loe and Kunii (1992) presented techniques of homotopy sweep.

Their methods are flexible in terms of shape definition, but the representations of surfaces are restricted to tensor form. That is to say, the surface expressions comprise only the product of two independent parameters for cross sectional curves and guide curves, and this property restricts the topology of surfaces.

3 Surface Model

In this section, we propose a method of generating an m-sided patch surrounded by m boundary curves by using m-variate coordinates. We first introduce generalized barycentric coordinates for the m-sided domain. Next, we generate two sweep surfaces by sweeping two cross sectional curves along the i^{th} boundary curve; the cross sectional curves are selected from those curves sharing an intersection with the i^{th} curve. Next, we generate a surface by blending the above two sweep surfaces, and call it the i^{th} *sub-surface*. Finally, we generate an m-sided patch by blending the m sub-surfaces. This blending uses a transfinite interpolant that preserves the geometric continuity of the i^{th} sub-surface on the i^{th} boundary curve.

3.1 Generalized barycentric coordinates

Let a patch be surrounded by boundary curves $\mathbf{C}_i$, $i = 1, 2, ..., m$, and be defined over an m-lateral polygon called a *domain polygon*. Each vertex of the domain polygon p_i corresponds to an intersection of the curves, and each edge of the domain polygon e_i corresponds to a section of a curve between two intersections, as shown in Figure 1.

We embed the multivariate coordinates on the domain polygon by using the generalized barycentric coordinates proposed by Loop and DeRose (1989; 1990). The mapping from each point p on the domain polygon $P = \{p_1, p_2, ..., p_m\}$ to the generalized barycentric coordinates $\ell = \{\ell_1, \ell_2, ..., \ell_m\}$ is defined as follows:

$$\ell_i(p) = \frac{\pi_i(p)}{\sum_{i=1}^{m} \pi_i(p)} \quad , \qquad \pi_i(p) = \frac{\prod_{i=1}^{m} \alpha_i(p)}{\alpha_{i-1}(p)\alpha_i(p)} \quad ,$$

where $\alpha_i(p)$ denote the signed area of triangle $p\, p_i\, p_{i+1}$, whose sign is determined to be positive if p is inside P.

The coordinates ℓ define m-sided patches, and have the following properties:

- *Division of one*: $\sum_{i=1}^{m} \ell_i = 1$.
- *Vertex preservation*: p_i is mapped to $\ell_i = 1 \cap \ell_{j \neq i} = 0$.
- *Edge preservation*: e_i is mapped to $\ell_i + \ell_{i+1} = 1 \cap \ell_{j \neq i, i+1} = 0$.
- *Pseudo-affine property*: $p = \sum_{i=1}^{m} \ell_i(p)\, p_i$ holds if the domain polygon is regular.

In this subsection, we propose a method of constructing sub-surfaces. The i^{th} sub-surface, denoted by $\mathbf{S}_i$, is generated from the boundary curves on e_i, e_{i-1}, and e_{i+1}. Without loss of generality, we assume that the curve $\mathbf{C}_i(t)$ on e_i spans from 0 to Δ_i, and let $\mathbf{C}_{i-1}(0)$ and $\mathbf{C}_{i+1}(0)$ coincide with the vertices p_i and p_{i+1} respectively (see Figure 1). All the curves can have arbitrary parametric forms that map the value of a parameter $t \in [0, \Delta_i]$ to a 3D point $\mathbf{C}_i(t)$; they can have arbitrary degrees and nodes of segments if they are represented by polynomial spline functions. The spans Δ_i are also arbitrarily set; however, it is desirable to make the spans be proportional to the arc length or the Euclidean distance between two intersections of the curve $\mathbf{C}_i$ in order to avoid generating unnaturally shaped surfaces.

We here consider a sweep of cross sectional curves $\mathbf{C}_{i-1}$ and $\mathbf{C}_{i+1}$ in which $\mathbf{C}_i$ is regarded as a guide curve.

First, we introduce the local parameters u_i and v_i: u_i defines the parameter space of the guide curve $\mathbf{C}_i$, and v_i defines that of the cross sectional curves $\mathbf{C}_{i-1}$ and $\mathbf{C}_{i+1}$.

$$\begin{aligned}
\mu &= \left\lfloor \frac{m}{2} \right\rfloor \quad , \quad \omega_i = \begin{cases} \Delta_i & ; \ m \ \text{even} \\ \Delta_i/(1-\ell_{i+\mu+1}) & ; \ m \ \text{odd} \end{cases} \quad , \\
u_i &= \omega_i \sum_{k=1}^{\mu} \ell_{i+k} \quad , \qquad (1) \\
v_i &= \left[\Delta_{i-1}\left(1-\frac{u_i}{\Delta_i}\right) + \Delta_{i+1}\frac{u_i}{\Delta_i} \right] \sum_{k=2}^{m-1} \ell_{i+k} \quad ,
\end{aligned}$$

where the suffix of ℓ is defined to modulus m, and $\lfloor \ \rfloor$ represents a floor function.

Let the cross directional derivative D_i about e_i be defined by partial derivatives with respect to ℓ_i, $i = 1, 2, ..., m$:

$$\begin{aligned}
D_i \; = \; & (\ell_i + \ell_{i-1}) \frac{\partial}{\partial \ell_{i-1}} + (\ell_{i+1} + \ell_{i+2}) \frac{\partial}{\partial \ell_{i+2}} \\
& + \sum_{k=3}^{m-2} \ell_{i+k} \frac{\partial}{\partial \ell_{i+k}} - (\Delta_i - u_i) \frac{\partial}{\partial \ell_i} - u_i \frac{\partial}{\partial \ell_{i+1}} \quad ,
\end{aligned}$$

then $(u_i \, , \, v_i)$ form an orthogonal parameter space with respect to D_i:

$$\begin{aligned}
D_i \, u_i &= 0 \, , \\
D_i \, v_i &= \Delta_{i-1}\left(1-\frac{u_i}{\Delta_i}\right) + \Delta_{i+1}\frac{u_i}{\Delta_i} \, .
\end{aligned}$$

The sweep surface $\mathbf{T}_{i,p}$ is then represented in $(u_i \, , \, v_i)$ coordinates by

$$\mathbf{T}_{i,p}(u_i \, , \, v_i) = \mathbf{M}_p \left[\mathbf{C}_p(v_i) - \mathbf{C}_p(0)\right] + \mathbf{C}_i(u_i) \, , \quad p = i-1, \ i+1 \, , \qquad (2)$$

where $\mathbf{M}_p$ represents an affine transformation matrix whose 3×3 elements are functions depending only on the parameter u_i, and a parallel (or translational) sweep of $\mathbf{C}_p$ along $\mathbf{C}_i$ is obtained by setting $\mathbf{M}_p$ to a unit matrix. The elements of the matrix $\mathbf{M}_p$ can be also determined by calculating an orthonormal coordinate frames along the guide curve $\mathbf{C}_i$, using a method of Frenet frame or rotation minimizing sweep (Klok, 1986). This method imposes only weak restrictions on the guide curve; it must be regular and twice continuously differentiable with non-vanishing curvature.

Next, we generate the i^{th} sub-surface $\mathbf{S}_i$ by blending the two sweep surfaces $\mathbf{T}_{i,i-1}$ and $\mathbf{T}_{i,i+1}$ with functions $g_{i-1}(u_i)$ and $g_{i+1}(u_i)$

$$\mathbf{S}_i(\ell) = g_{i-1}(u_i)\, \mathbf{T}_{i,i-1}(u_i \, , \, v_i) + g_{i+1}(u_i)\, \mathbf{T}_{i,i+1}(u_i \, , \, v_i) \, , \qquad (3)$$

where g_{i-1} and g_{i+1} are determined in such a way as to satisfy the following constraints:

$$\begin{aligned}
g_{i-1}(u_i) + g_{i+1}(u_i) &\equiv 1 \, , \quad u_i \in [0, \Delta_i] \, , \\
g_{i-1}(0) = g_{i+1}(\Delta_i) &= 1 \, . \qquad (4)
\end{aligned}$$

We here introduce weight parameters w_i for each curve $\mathbf{C}_i$, and construct the functions g_{i-1} and g_{i+1} by using w_i as

$$\begin{aligned}
g_{i-1}(u_i) &= \frac{w_{i-1}(\Delta_i - u_i)}{w_{i-1}(\Delta_i - u_i) + w_{i+1} u_i} \, , \\
g_{i+1}(u_i) &= \frac{w_{i+1} u_i}{w_{i-1}(\Delta_i - u_i) + w_{i+1} u_i} \, ,
\end{aligned}$$

where the parameter w_i controls the influence of $\mathbf{C}_i$ on the shape of the sub-surface $\mathbf{S}_i$.

The values of the elements of the matrix $\mathbf{M}_p$ and the function g_p are uniquely determined for each pair of the intersecting curves $(\mathbf{C}_i \, , \, \mathbf{C}_p)$. The above-mentioned methods of determining these values are effective in that they can calculate smooth and natural shaped surfaces fast and stably. We may use nonlinear optimization techniques to minimize the variation of curvature or the energy of surfaces in order to generate the fair shapes (Moreton and Séquin, 1992). Their calculation, however, is time-consuming and unstable for our surface model.

It is noteworthy that the sub-surface $\mathbf{S}_i$ defined by the above expressions has the same order of geometric continuity as the cross sectional curves $\mathbf{C}_{i-1}$ and $\mathbf{C}_{i+1}$ (see Appendix).

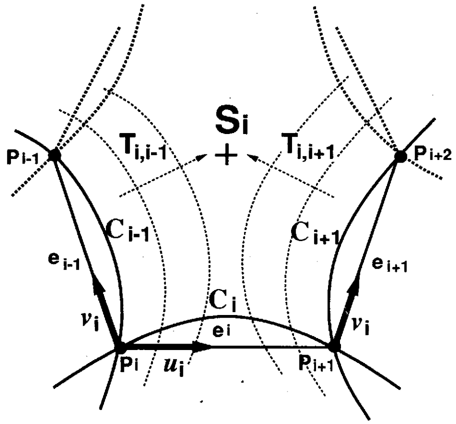


Figure 1: Sweeping of curves

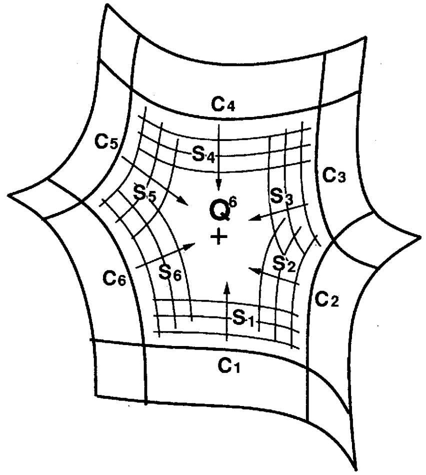


Figure 2: Blending of sub-surfaces

3.3 Blending of sub-surfaces

In this subsection, we generate an m-sided patch $\mathbf{Q}^m$ by introducing a *blending function* B_i^n of the n^{th}-degree. The patch $\mathbf{Q}^m$ is composed by the convex combination of the m sub-surfaces $\mathbf{S}_i$, $i = 1, 2, ..., m$ as follows (see Figure 2):

$$\mathbf{Q}^m(\ell) = \sum_{i=1}^{m} \mathbf{S}_i(\ell)\, B_i^n(\ell) \,,$$

where B_i^n is defined by

$$B_i^n(\ell) = \frac{(\ell_i \ell_{i+1})^n}{\sum_{k=1}^{m} (\ell_k \ell_{k+1})^n} \,. \tag{5}$$

Equation (5) imposes the following conditions on the edges:

$$\begin{aligned} B_i^n(\ell) &= 1 \,; \quad \ell \in e_i \,, \\ B_i^n(\ell) &= 0 \,; \quad \ell \in e_{j \neq i} \,, \\ \frac{\partial^h B_i^n(\ell)}{\partial \ell_j{}^h} &= 0 \,; \quad \ell \in e_i \,, \\ & i \,,\, j = 1, 2, ..., m \,,\quad h = 1, 2, ..., n-1 \,, \end{aligned}$$

where $\ell \in e_i := \{\ell_i + \ell_{i+1} = 1 \cap \ell_{j \neq i, i+1} = 0\}$. Consequently, $\mathbf{Q}^m$ preserves the derivatives of $\mathbf{S}_i$ with respect to D_i , up to the $(n-1)^{\text{th}}$-order on e_i:

$$D_i^h \mathbf{Q}^m(\ell) = D_i^h \mathbf{S}_i(\ell) \;\;;\;\; \ell \in e_i \,,\;\; h = 0, 1, ..., n-1 \,. \tag{6}$$

Equation (6) implies that the continuity condition of $\mathbf{Q}^m$ on e_i is reduced to that of $\mathbf{S}_i$. Also, patch $\mathbf{Q}^m$ has twist compatibility at the vertices (or corners) of the domain polygon, where compatibility was introduced by Gregory (1974) for a rectangular patch.

The function B_i^n has singular points on the corners, but the singularities can be removed by adopting the limiting behavior of B_i^n near the vertices such that

$$B_i^n(\ell) = \begin{cases} 0 \;\;; & \ell \in p_{j \neq i \,,\, i+1} \\ 1/2 \;\;; & \ell \in p_{j = i \,,\, i+1} \end{cases} \,,$$

where $\ell \in p_i := \{\ell_i = 1 \cap \ell_{j \neq i} = 0\}$. Note that these limiting values preserve the continuity of the generated surface.

This blending function is regarded as a generalization of the interpolant proposed by Nielson (1987) and Hagen and Pottmann (1989), whose methods concern a triangular domain. Charrot and Gregory (1984) proposed a blending function that has a similar property. Their function, however, uses a combination of three successive variables for pentagonal patches and interpolates the values on two sides, whereas our function uses a combination of two successive variables and interpolates on one side. Notice that the multivariate coordinates in the Gregory-Charrot scheme are defined by the perpendicular distances of a point from the sides of a regular polygon, and are thus not identical with the generalized barycentric coordinates.

In Figure 3, we show the equi-valued line plots of B_i^3 for domain polygons with three, four, and five sides, where each line indicates $n/10$, $n = 0, 1, ..., 10$.

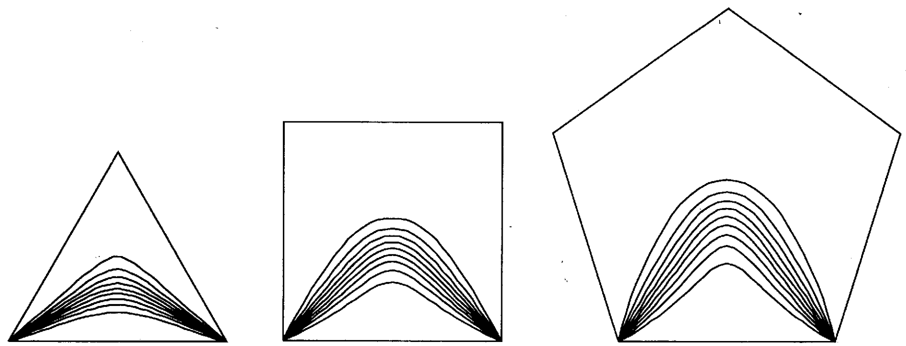


Figure 3: Equi-valued line plots of B_i^3

4 Modifications for singular topology

The sub-surface $\mathbf{S}_i$ described in Subsection 3.2 destroys the condition of geometric continuity with an adjacent sub-surface $\bar{\mathbf{S}}_i$ along the i^{th} boundary, if the constituent cross sectional curves for $\mathbf{S}_i$ and $\bar{\mathbf{S}}_i$ are not successively parameterized.

An irregular network of curves causes such discontinuity of cross sectional curves between adjacent sub-surfaces if the following conditions on an intersection are satisfied:

- More than two boundary curves intersect at a common vertex (called, *multiple intersection*).
- The open end of a curve intersects in the middle of the other curve (called, *T-connected intersection*).

Besides, the method in Subsection 3.2 cannot deal with such conditions on a domain as

- An open set of the boundary curves defines the domain of a patch (called, an *open-sided patch*).
- Only two boundary curves enclose the domain of a patch (called, a *two-sided patch*).

We call the above four conditions *singular topology*.

We consider that a curve network of singular topology is necessary in order to design complicated shapes flexibly, and therefore modify the rules of generating and blending sub-surfaces so that they satisfy geometric continuity for singular topology. The following four subsections explain the modified methods for each condition of singular topology.

4.1 Multiple intersection

More than two curves often intersect at a common point in a network; this point may represent a pole of a sphere or the center of a symmetrical shape.

Let the guide curve $\mathbf{C}_i$ have a multiple intersection with two curves $\mathbf{C}^1_{i-1}$ and $\mathbf{C}^2_{i-1}$, as shown in Figure 4 (a). The adjacent sub-surfaces $\mathbf{S}_i$ and $\bar{\mathbf{S}}_i$ must have a boundary curve that is successively parameterized; however, two cross sectional curves $\mathbf{C}^1_{i-1}$ and $\mathbf{C}^2_{i-1}$ are independently parameterized. We therefore replace the cross sectional curve for $\mathbf{S}_i$ and $\bar{\mathbf{S}}_i$ with a common curve $\hat{\mathbf{C}}_{i-1}$ that is continuous at the multiple intersection. For example, $\hat{\mathbf{C}}_{i-1}$ is constructed by averaging the cros sectional curves such that:

$$\hat{\mathbf{C}}_{i-1}(t) = \frac{\sum_{k=1}^{\lambda} \mathbf{C}^k_{i-1}(t)}{\lambda},$$

where λ indicates the number of the curves $\mathbf{C}^k_{i-1}$ that intersect with C at the multiple intersection.

This modification of cross sectional curves ensures that the sub-surface $\mathbf{S}_i$ and $\bar{\mathbf{S}}_i$ have geometric continuity of the same order as the curves $\mathbf{C}^k_{i-}$ However, the final surface does not satisfy geometric continuity at th multiple intersection, because of inconsistency of the geometric quantitie at the intersections. Nevertheless, we can modify the intersecting curve in such a way that geometric continuity is satisfied at the multipl intersection. For G^1 continuity, the first-order derivatives of the curves a adjusted so that they are on a common plane at the multiple intersectio For G^2 continuity, the second-order derivatives of the curves are als modified so that the curves have a consistent principal curvature principal direction at the multiple intersection (Miura and Wang, 1992

4.2 T-connected intersection

Hierarchical representation of a curve network is effective for designir complicated shapes, and it is realized by using T-connected intersectio in the network.

Let an open end of a curve $\mathbf{C}_T$ be connected to the middle of a bounda curve $\mathbf{C}_i(u)$ at $u = r_i\,\Delta_i$, and let the curve $\mathbf{C}_T$ split a sub-surface $\mathbf{S}_i$ int two sub-surfaces $\mathbf{S}^0_i$ and $\mathbf{S}^1_i$, as shown in Figure 4 (b).

It is obvious that the sub-surfaces $\mathbf{S}^0_i$ and $\mathbf{S}^1_i$ cannot have geometr continuity with the adjacent sub-surfaces $\bar{\mathbf{S}}_i$ if they are constructed b using $[\mathbf{C}^0_{i-1}, \mathbf{C}^0_i, \mathbf{C}^0_{i+1}]$ and $[\mathbf{C}^1_{i-1}, \mathbf{C}^1_i, \mathbf{C}^1_{i+1}]$ respectively.

We generate sub-surfaces that have geometric continuity with $\bar{\mathbf{S}}_i$ a follows:

1. Generate sub-surface $\mathbf{S}_i$ by neglecting $\mathbf{C}_T$ and using $\mathbf{C}^0_{i-1}$ and $\mathbf{C}^1_{i-}$ as cross sectional curves and $\mathbf{C}_i := \mathbf{C}^0_i \cup \mathbf{C}^1_i$ as a guide curve.
2. Split the sub-surface $\mathbf{S}_i(u_i, v_i)$ at $u_i = r_i\,\Delta_i$ into two sub-surface $\mathbf{S}^0_i(u_i, v_i) := \mathbf{S}_i(r_i\,u_i, v_i)$, $\mathbf{S}^1_i(u_i, v_i) := \mathbf{S}_i((1-r_i)\,u_i + r_i\,\Delta_i, v_i$

The curve $\mathbf{C}_T$ is used as a guide curve in constructing the sub-surfac $\mathbf{S}^0_{i+1}$ and $\mathbf{S}^1_{i-1}$ and used as a cross sectional curve for $\mathbf{S}^0_{i+2}$ and $\mathbf{S}^1_{i-2}$. As result, the final surfaces obtained by blending sub-surfaces have geometr continuity with $\bar{\mathbf{S}}_i$, and they exactly interpolate the curve $\mathbf{C}_T$ because the sub-surfaces $\mathbf{S}^0_{i+1}$ and $\mathbf{S}^1_{i-1}$ contain the curve $\mathbf{C}_T$.

4.3 *Open-sided patch*

When the shape of a surface is being designed, a transitional network of curves may contain a domain that is topologically not closed. We here consider a modification of the method described in Section 3 for generating a surface from a set of boundary curves surrounding the open domain. This technique allows designers to check the shape of the transitional network.

Assume that a virtual curve $\mathbf{C}_m$ is added to a set of boundary curves $\mathbf{C}_i$, $i = 1, 2, ..., m-1$ in order to surround an m-sided domain, as shown in Figure 4 (c).

Because sweep surfaces $\mathbf{T}_{1,m}$ and $\mathbf{T}_{m-1,m}$ cannot be defined, sub-surfaces $\mathbf{S}_1$ and $\mathbf{S}_{m-1}$ are equated to $\mathbf{T}_{1,2}$ and $\mathbf{T}_{m-1,m-2}$ respectively. We then generate the m-sided patch $\mathbf{Q}^m$ by blending the sub-surfaces as follows:

$$\mathbf{Q}^m(\ell) = \sum_{i=1}^{m-1} \mathbf{S}_i(\ell)\, \hat{B}^n_i(\ell) ,$$

$$\hat{B}^n_i(\ell) = \frac{(\ell_i \ell_{i+1})^n + \kappa\,(\ell_1 \ell_m)^n}{\sum_{k=1}^{m} (\ell_k \ell_{k+1})^n} , \quad \kappa = \begin{cases} u_m & ; \quad i = 1 \\ 1 - u_m & ; \quad i = m-1 \\ 0 & ; \quad i \neq 1,\ m-1 \end{cases} .$$

4.4 *Two-sided patch*

The definition of a domain polygon in Subsection 3.1 implies that a two-sided patch cannot be defined; however, a domain enclosed by two curves often occurs in the construction of a curve network, especially in the first stage of designing a shape. Therefore, we propose a method of skinning the two-sided domain by extending it into a quadrilateral region, as shown in Figure 4 (d).

We here split the curves $\mathbf{C}_1$ and $\mathbf{C}_2$ into two pieces as

$$\mathbf{C}^0_1(t) := \mathbf{C}_1\left(\frac{t}{2}\right) , \quad \mathbf{C}^1_1(t) := \mathbf{C}_1\left(\frac{\Delta_1 + t}{2}\right) ,$$

$$\mathbf{C}^0_2(t) := \mathbf{C}_2\left(\frac{t}{2}\right) , \quad \mathbf{C}^1_2(t) := \mathbf{C}_2\left(\frac{\Delta_2 + t}{2}\right) .$$

Then the sub-surface $\mathbf{S}_i$, $i = 1, 2, 3, 4$ is constructed by the metho proposed in Subsection 3.2 with the split curves. The sub-surface $\mathbf{S}_1$ i constructed by using $\mathbf{C}^0_1$ as a guide curve, and by using $\mathbf{C}^0_2$ and $\mathbf{C}^1_2$ a cross sectional curves, and the sub-surface $\mathbf{S}_2$ is constructed with $\mathbf{C}^1_1$, C and $\mathbf{C}^1_2$. The sub-surface $\mathbf{S}_3$ and $\mathbf{S}_4$ are similarly constructed by using C and $\mathbf{C}^1_2$ as a guide curve respectively, and by using $\mathbf{C}^0_1$ and $\mathbf{C}^1_1$ as cros sectional curves. This construction ensures the geometric continuity c the sub-surfaces $\mathbf{S}_i$ on e_i .

The final four-sided patch $\mathbf{Q}^4$ is generated by using the same blendin functions B^n_i introduced in Subsection 3.3 as

$$\mathbf{Q}^4(\ell) = \sum_{i=1}^{4} \mathbf{S}_i(\ell)\, B^n_i(\ell) .$$

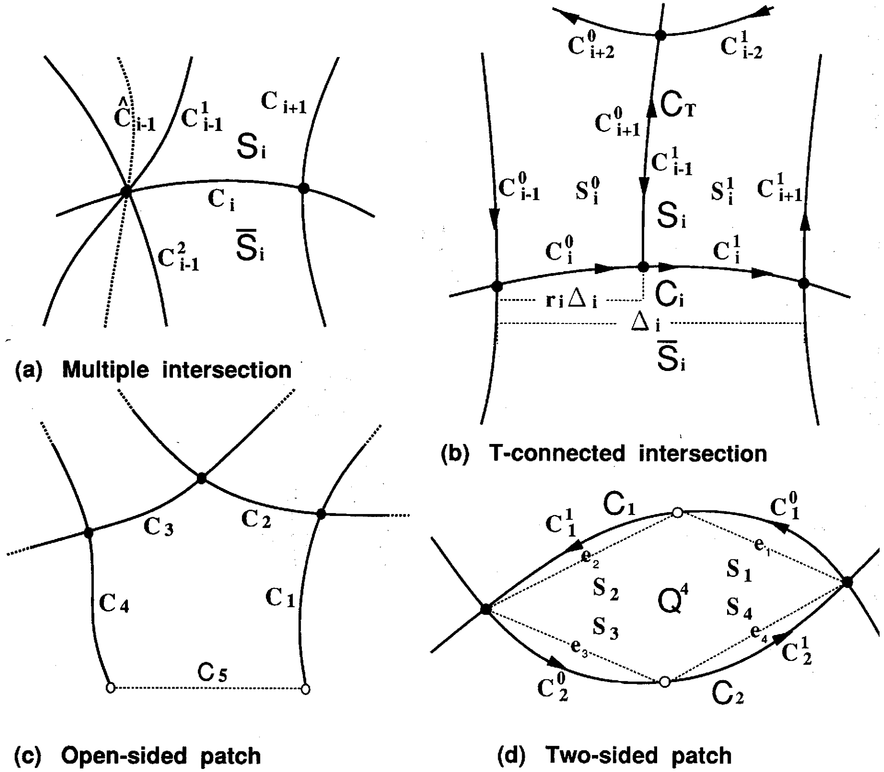


Figure 4: Singular topology

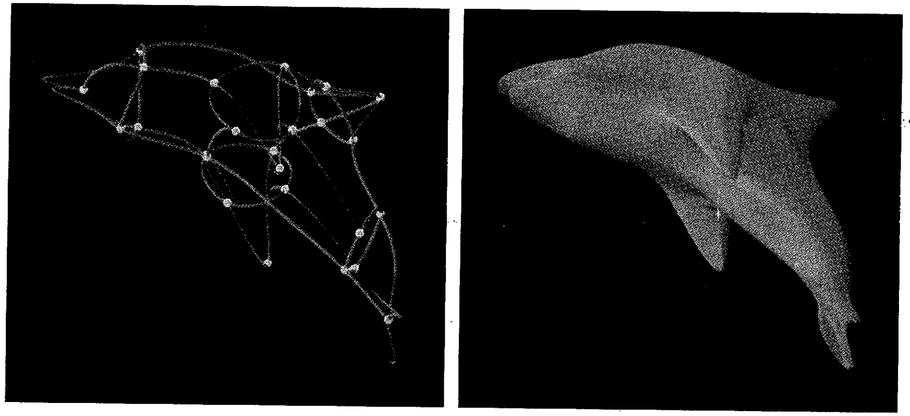
5 Examples

In this section, we present some examples of curve networks and surfaces generated from them by our method.

Figure 5 shows an example of a closed surface generated by a network that excludes the singular topology described in Section 4, where yellow balls indicate intersections of curves.

In Figure 6, we show an example of a surface defined by a curve network that contains multiple and T-connected intersections, where green balls indicate multiple intersections and red balls indicate T-connected intersections, and Figure 7 shows an example of a surface defined by a curve network that has open-sided and two-sided domains.

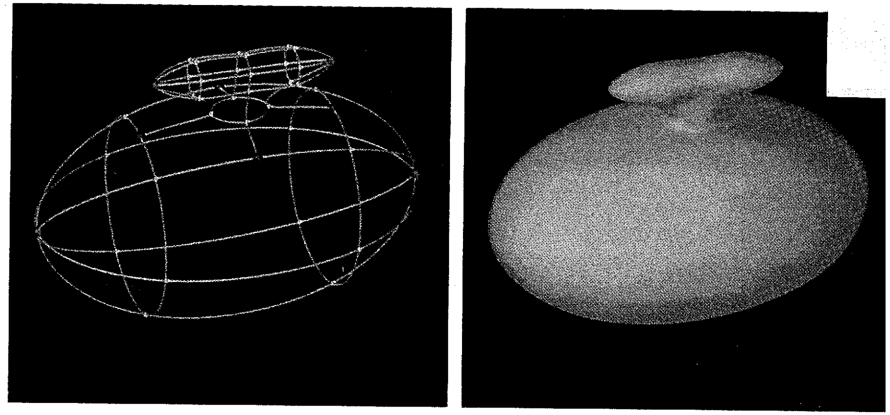
All surfaces are generated from curves that have C^2 continuity represented by cubic splines, where the weight parameters w_i are set to 1 for all curves, so that every pair of sweep surfaces are blended linearly. Translational sweeps are adopted for all the examples by setting $\mathbf{M}_p$ to a unit matrix. Surface data are generated by tessellating m-sided patches into triangular-stripes, and are rendered by using the Phong shading method on an IBM RISC System/6000 [1].



(a) A network **(b) A surface**

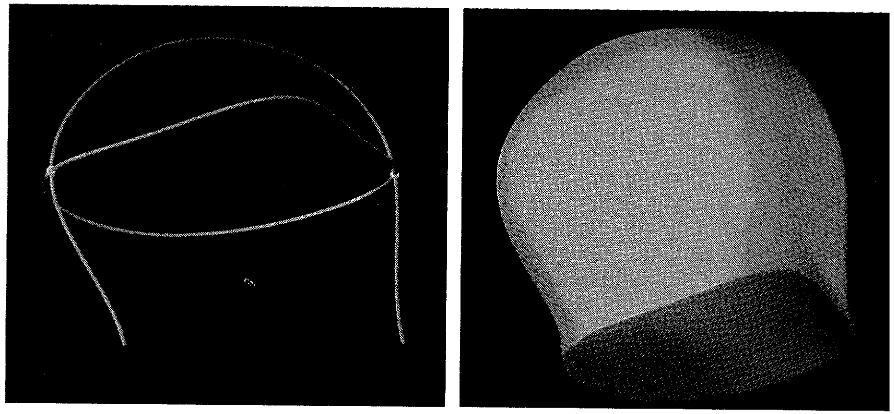
Figure 5: A network excluding singular topology

[1] IBM RISC System/6000 is a trademark of IBM Corp.



(a) A network **(b) A surface**

Figure 6: A network including multiple and T-connected intersections



(a) A network **(b) A surface**

Figure 7: A network including open-sided and two-sided domains

6 Conclusions

We have presented a method of generating surfaces from a network of curves. Compared with existing methods, it has the following advantages:

- Surfaces can be defined by a network of curves that have arbitrary parametric forms.
- The network can be defined on an arbitrary connective topology, and all domains surrounded by the curves can be skinned.
- Each patch can have G^n continuity with adjacent patches if all curves surrounding it have C^n continuity.

These advantages make it possible to construct a surface model on user-friendly CAD interfaces for designing shapes by inputting 3D curves, such as (1) a sketching system with pen-input device, or (2) a virtual environment system with an advanced 3D input device such as a Spaceball or a DataGlove.

We have not considered the topological representation of a network in this paper. Our method assumes that the topological data for the network is predetermined. Because of the complexity of the topology in an irregular network, it is difficult to construct topological data automatically from curve data. The topological representation and operations (including non-manifold conditions) for the network are our next research topics.

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