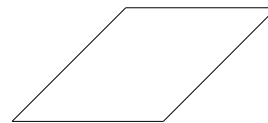# Exercise Computer graphics – (till Mai 19, 2009)

Rotations

Exercise 18:

a) It is possible to decompose rotations into a number of succeeding shears. What is the least number of shears a rotation in 2D can be decomposed into? Explicitly state which shears you need.

b) In which way does an image manipulation program benefit from the decomposition you suggested above?
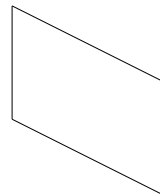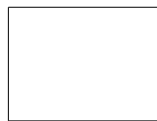
horizontal shear =>
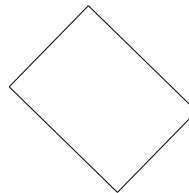$$\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

vertical shear =>
$$\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}$$

rotation =>
$$\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

Exercise Computer graphics

Rotations

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b) In which way does an image manipulation program benefit from the decomposition you suggested above?

Solution b): The image can be rotated by shifting data within memory only.

Express a 2D Rotation as 3 shears

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+st & s \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+st & u+stu+s \\ t & ut+1 \end{pmatrix}$$

I $\quad 1+st = \cos\alpha$

II $\quad u + s(tu+1) = -\sin\alpha$

III $\quad t \qquad = \sin\alpha \quad \Rightarrow \quad \underline{t=\sin\alpha}$

IV $\quad ut+1 \qquad = \cos\alpha$

III into I: $\quad 1 + s\cdot\sin\alpha = \cos\alpha$

$$s = \frac{\cos\alpha - 1}{\sin\alpha}$$

III into IV: $\quad u\cdot\sin\alpha + 1 = \cos\alpha$

$$u = \frac{\cos\alpha - 1}{\sin\alpha}$$

check II:

$$\frac{\cos\alpha - 1}{\sin\alpha} + \frac{\cos\alpha - 1}{\sin\alpha}\left(\sin\alpha \ \frac{\cos\alpha}{\sin\alpha} + 1\right) = -\sin\alpha \quad / \cdot \sin\alpha$$

$$\cos\alpha - 1 + \cos^2\alpha - \cos\alpha = -\sin^2\alpha$$

$$\cos^2\alpha - 1 = -\sin^2\alpha$$

$$\cos^2\alpha + \sin^2\alpha = 1 \quad \checkmark$$