

## Exercise Computer graphics – (till May 5, 2009)

### Bicubic/Bilinear surfaces

Exercise 15: We have seen how bi-cubic patches and planar patches of 1<sup>st</sup> degree were defined in the lecture.

- a) Define a patch of 2<sup>nd</sup> degree. The tangent on one side of the patch should be definable explicitly.

Solution:

$Q(t) = a + bt + ct^2$   
 $Q'(t) = b + 2ct$

$Q(0) = P_1(s) = a$   
 $Q(1) = P_2(s) = a + b + c$   
 $Q'(0) = R(s) = b$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} P_1(s) \\ P_2(s) \\ R(s) \end{pmatrix}$$

$$(1 \ t \ t^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} P_1(s) \\ P_2(s) \\ R(s) \end{pmatrix} = Q(t)$$

How are  $P_1(s)$ ,  $P_2(s)$  and  $R(s)$  defined?

$$P_1(s) = (1 \ s \ s^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} k_{11} \\ k_{21} \\ k_{11} - k_{01} \end{pmatrix}; \quad P_2(s) = (1 \ s \ s^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} k_{12} \\ k_{22} \\ k_{12} - k_{02} \end{pmatrix}$$

$$R(s) = (1 \ s \ s^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} k_{11} - k_{10} \\ k_{21} - k_{20} \\ (k_{11} - k_{10}) - (k_{01} - k_{00}) \end{pmatrix}$$

Hint:  $(ABC)^T = C^T B^T A^T$

$$Q(s,t) = (1 \ t \ t^2) \begin{pmatrix} 1 & 0 & 0 & k_{11} & k_{21} & (k_{11} - k_{01}) \\ 0 & 0 & 1 & k_{12} & k_{22} & (k_{12} - k_{02}) \\ -1 & 1 & -1 & (k_{11} - k_{10}) & (k_{21} - k_{20}) & [(k_{11} - k_{10}) - (k_{01} - k_{00})] \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ s \\ s^2 \end{pmatrix}$$

Interpretation of the geometry matrix above:

$$\begin{pmatrix} Q(0,0) & Q(1,0) & \frac{\partial Q(0,0)}{\partial s} \\ Q(0,1) & Q(1,1) & \frac{\partial Q(0,0)}{\partial s} \\ \frac{\partial Q(0,0)}{\partial t} & \frac{\partial Q(1,0)}{\partial t} & \frac{\partial Q(0,0)}{\partial s \partial t} \end{pmatrix}$$

## Exercise Computer graphics – (till November 13, 2007)

### Bicubic/Bilinear surfaces

Exercise 15: We have seen how bi-cubic patches and planar patches of 1<sup>st</sup> degree were defined in the lecture.

- b) Interpret the influence of all components of the geometry matrix on the Patch.

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