Exercise Computer graphics

Line clipping according to Cyrus Beck – (till March 31, 2009)

Exercise 10: Line clipping

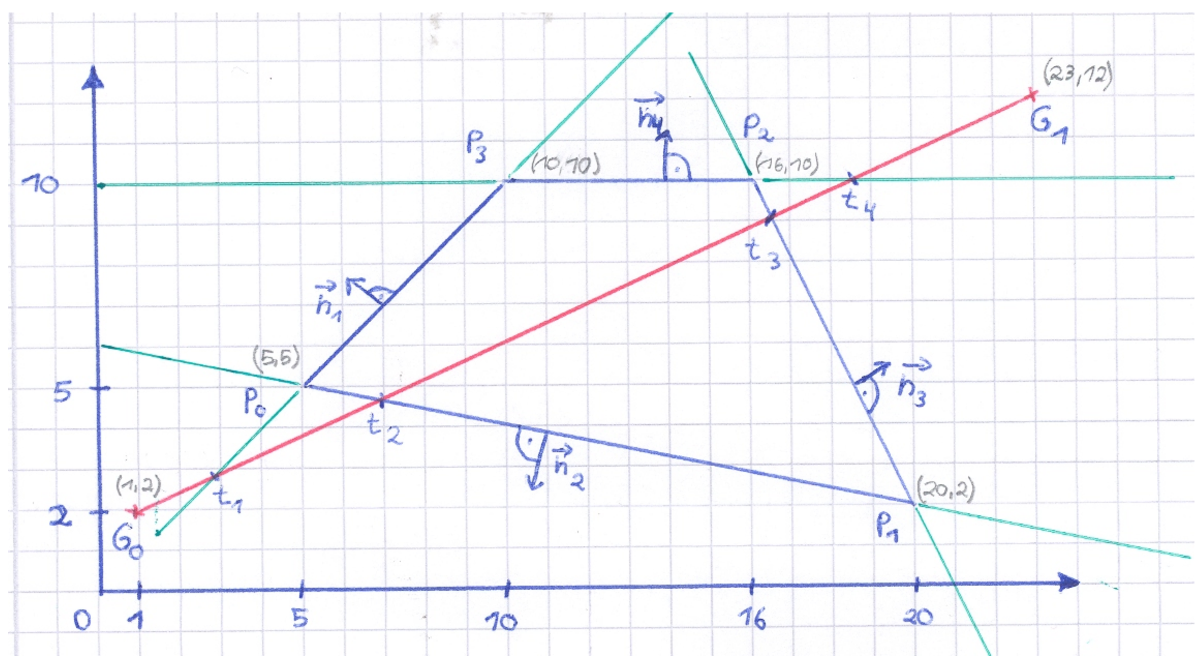
Let a clipping polygon be defined by the vertices
(5, 5), (20, 2), (16, 10), (10, 10)

and a line between
(1,2) and (23, 12)

(a) Perform the Cyrus Beck clipping algorithm. Find out for each intersection parameter t whether it is "entering" or "leaving" and finally determine which parameters for t are of interest only.

(b)

In the general case of an n-sided polygon: How many intersections have to be performed at most for every line to be displayed?



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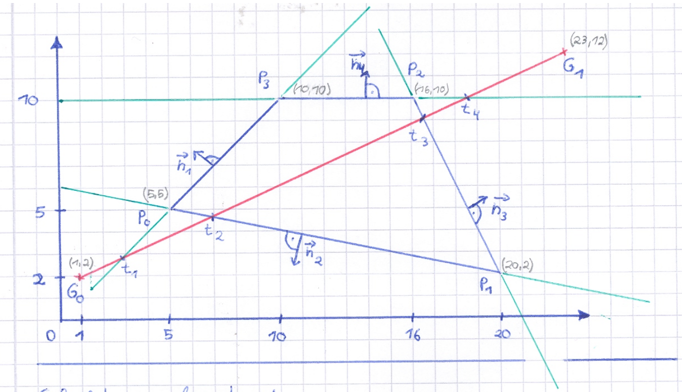
Solution (1):

LINE CLIPPING according to CYRUS BECK



Calculate normal vectors:

$\vec{n}_1 = \begin{pmatrix}-1\\1\end{pmatrix}$; $\vec{n}_4 = \begin{pmatrix}0\\1\end{pmatrix}$

$\vec{n}_2 (P_1 - P_0) \stackrel{!}{=} 0 \Rightarrow \begin{pmatrix}n_{2}x\\n_{2}y\end{pmatrix}\begin{pmatrix}20-5\\2-5\end{pmatrix} = 0 \Rightarrow 15 n_2 x - 3 n_2 y = 0$

e.g.: $15(-1) - 3(-5) = 0$

$\Rightarrow n_2 = \begin{pmatrix}-1\\-5\end{pmatrix}$

Hint: The normal vectors do not require $|n| = 1$

$\vec{n}_3 (P_2 - P_1) \stackrel{!}{=} 0 \Rightarrow \begin{pmatrix}n_3 x\\n_3 y\end{pmatrix}\begin{pmatrix}16-20\\10-2\end{pmatrix} = 0 \Rightarrow -4 n_3 x + 8 n_3 y = 0$

e.g.: $-4\cdot(+2) + 8\cdot 1 = 0$

$\Rightarrow n_3 = \begin{pmatrix}2\\1\end{pmatrix}$

Intersection: $[G_0 + t(G_1 - G_0) - P_0]\, \vec{n}_1 \stackrel{!}{=} 0$

$$\Rightarrow t = \frac{P_0 \vec{n}_1 - G_0 \vec{n}_1}{\vec{n}_1 (G_1 - G_0)} = \frac{\vec{n}_1 (P_0 - G_0)}{\vec{n}_1 (G_1 - G_0)}$$

$$t_1 = \frac{\begin{pmatrix}-1\\1\end{pmatrix}\left[\begin{pmatrix}5\\5\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right]}{\begin{pmatrix}-1\\1\end{pmatrix}\left[\begin{pmatrix}23\\12\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right]} = \frac{-4+3}{-22+10} = \frac{-1}{-12}$$ ← denominator < 0 ⇒ "entering"

$$t_2 = \frac{\begin{pmatrix}-1\\-5\end{pmatrix}\left[\begin{pmatrix}5\\5\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right]}{\begin{pmatrix}-1\\-5\end{pmatrix}\begin{pmatrix}22\\10\end{pmatrix}} = \frac{-4-15}{-22-50} = \frac{-19}{-72}$$ ← "entering"

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Solution (2):

$$t_3 = \frac{\begin{pmatrix}2\\1\end{pmatrix}\left[\begin{pmatrix}20\\2\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right]}{\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}22\\10\end{pmatrix}} = \frac{38}{44+10} = \frac{19}{27}$$ ← denominator > 0 ⇒ „leave"

$$t_4 = \frac{\begin{pmatrix}0\\1\end{pmatrix}\left[\begin{pmatrix}16\\10\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}\right]}{\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}22\\10\end{pmatrix}} = \frac{8}{10}$$ ← denominator > 0 ⇒ „leave"

We are interested in the largest entering value and the smallest leaving value

$$\Rightarrow t_E = \frac{19}{72} \; ; \; t_L = \frac{19}{27}$$

$$\Rightarrow P_E = \begin{pmatrix}1\\2\end{pmatrix} + \frac{19}{72}\begin{pmatrix}22\\10\end{pmatrix} \approx \begin{pmatrix}6{,}8\\4{,}6\end{pmatrix}$$

$$P_L = \begin{pmatrix}1\\2\end{pmatrix} + \frac{19}{27}\begin{pmatrix}22\\10\end{pmatrix} \approx \begin{pmatrix}16{,}5\\9\end{pmatrix}$$

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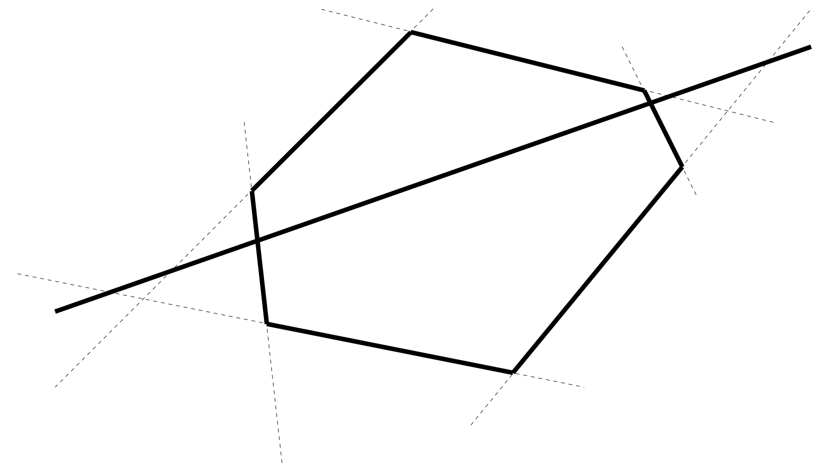


(b)

In the general case of an n-sided polygon: How many intersections have to be performed at most for every line to be displayed?

Solution:

Every line of the polygon can potentially intersect a line to be drawn, unless they are parallel.

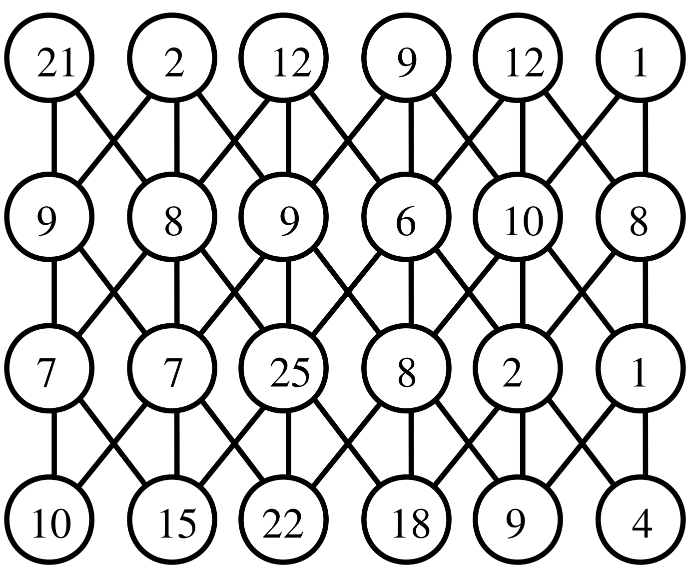
Exercise Computer graphics

Seam carving

Exercise 11:

The graph below displays a small image with gray values in the knots.

Find the optimal path from the upper line of pixels to the lower one which traverses as small gray value changes as possible. A gray values change is simply obtained as the absolute difference between two neighboring knots. The neighborhood relationships are denoted by the edges connecting the knots.



Exercise Computer graphics

Seam carving

Exercise 11:

Solution:

(see dashed circles)