

Exercise Computer graphics – (till March 17, 2009)

Ultra-fast line drawing

Exercise 6: In the lecture we have seen the mid-point algorithm for ellipses, however, only the part of the ellipse between 45° and 90° was drawn. Finish the arc between 0° and 45° and include it into the application.

Solution:

Handwritten notes for the midpoint algorithm for ellipses, showing the derivation of the update equations for the first and second octants.

Top left: $F(a, 0) = 0$

Equation for $(a - 0, 5, 1)$:

$$\begin{aligned} F(a - 0, 5, 1) &= b^2(a - 0, 5)^2 + a^2(1)^2 - a^2b^2 \\ &= b^2(a^2 - a + 0, 25) + a^2 - a^2b^2 \\ &= b^2a^2 - b^2a + 0,25b^2 + a^2 - a^2b^2 \\ &= a^2 + b^2(0,25 - a) \end{aligned}$$

Annotation: $(a - 0, 5, 1)$ or $(xp - 0, 5, yp + 1)$

Left side: $1. \text{ case } d < 0 \Rightarrow \text{choose } N \Rightarrow y++;$

Diagram: A coordinate system showing an ellipse centered at $(a, 0)$. A point on the ellipse is labeled (x_0, y_0) . A line segment connects (x_0, y_0) to the next point $(x_0 + 1, y_0 + 1)$, which is marked with a small circle. An arrow points from the text "choose $N \Rightarrow y++;$ " to this segment.

Bottom left: $d_{\text{new}} = F(x_0 - 0, 5, y_0 + 1)$

Equation for $(x_0 - 0, 5, y_0 + 1)$:

$$\begin{aligned} d_{\text{new}} &= F(x_0 - 0, 5, y_0 + 1) \\ &= b^2(x_0 - 0, 5)^2 + a^2(y_0 + 1)^2 - a^2b^2 \\ &= b^2x_0^2 - b^2x_0 + b^2(0,25) + a^2y_0^2 + 2a^2y_0 + a^2 - a^2b^2 \\ &= F(x_0 - 0, 5, y_0 + 1) + 2a^2y_0 + 3a^2 \end{aligned}$$

Bottom right: $\left[\begin{array}{l} F(x_0 - 0, 5, y_0 + 1) \\ = b^2(x_0 - 0, 5)^2 + a^2(y_0 + 1)^2 - a^2b^2 \\ = b^2x_0^2 - b^2x_0 + b^2(0,25) + a^2y_0^2 + 2a^2y_0 + a^2 - a^2b^2 \\ = (b^2x_0^2 - b^2x_0 + 0,25b^2) + (a^2y_0^2 + 2a^2y_0 + a^2 - a^2b^2) \end{array} \right] ;$

Bottom left: $2. \text{ case } d > 0 \Rightarrow \text{choose } NW \Rightarrow x--; y++;$

Equation for $(x_0 - 1, 5, y_0 + 1)$:

$$\begin{aligned} d_{\text{new}} &= F(x_0 - 1, 5, y_0 + 1) \\ &= b^2(x_0 - 1, 5)^2 + a^2(y_0 + 1)^2 - a^2b^2 \\ &= b^2x_0^2 - 3b^2x_0 + b^2(0,25) + a^2y_0^2 + 4a^2y_0 + a^2 - a^2b^2 \\ &= F(x_0 - 0, 5, y_0 + 1) + (-2b^2x_0) + 2b^2 + 2a^2y_0 + 3a^2 \\ &= F(x_0 - 0, 5, y_0 + 1) + b^2(2 - 2x_0) + a^2(2y_0 + 3) \end{aligned}$$

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Exercise 7: Reduce the ellipse-version of the mid-point algorithms to circles.
Why is this version faster?

Solution:

Circle equation:

$$\begin{aligned}x^2 + y^2 &= r^2 \\ \Rightarrow F(x, y) &= x^2 + y^2 - r^2\end{aligned}$$

Initialization: $P_0 = (0, r)$

Pixel $(0, r)$ must lie on the circle

$$\text{Thus } F(0, r) = 0^2 + r^2 - r^2 = 0$$

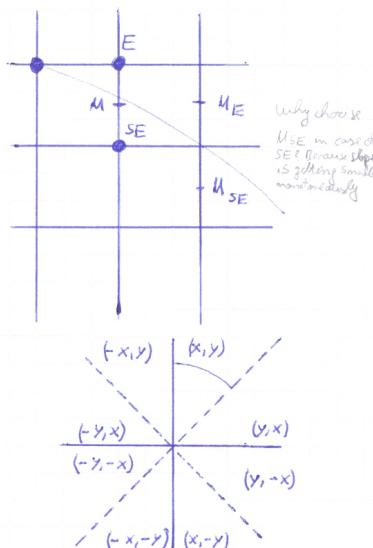
For the first M it holds true that $M = (1, r - 1/2)$

$$\begin{aligned}F(1, r - 1/2) &= 1^2 + (r - 1/2)^2 - r^2 = \\ &= 1 + r^2 - r + 0,25 - r^2 \\ &= 1,25 - r = d_{old}\end{aligned}$$

1. Fall: $d_{old} < 0 \Leftrightarrow M$ below circle so
choose $E \Rightarrow x = x + 1$

$$M_E = (x_0 + 2, y_0 + 0,5)$$

$$\begin{aligned}d_{new} &= F(x_0 + 2, y_0 + 0,5) = (x_0 + 2)^2 + (y_0 + 0,5)^2 - r^2 \\ &= x_0^2 + 4x_0 + 4 + y_0^2 + y_0 + 0,25 - r^2 \\ &= F(x_0 + 1, y_0 + 0,5) + 2x_0 + 3 \\ &\quad \boxed{\begin{aligned}F(x_0 + 1, y_0 + 0,5) \\ = (x_0 + 1)^2 + (y_0 + 0,5)^2 - r^2 \\ = x_0^2 + 2x_0 + 1 + y_0^2 + y_0 + 0,25 - r^2\end{aligned}}\end{aligned}$$



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Solution: (continued)

2. case: $d_{\text{old}} > 0 \Leftrightarrow$ in a circle so
choose SE $\Rightarrow x = x + 1; y = y - 1$

$$M_{SE} = (x_0 + 2, y_0 - 1, 5)$$

$$\begin{aligned} d_{\text{new}} &= F(x_0 + 2, y_0 - 1, 5) = (x_0 + 2)^2 + (y_0 - 1, 5)^2 - r^2 \\ &= (x_0^2 + 4x_0 + 4) + y_0^2 - 3y_0 + 2,25 - r^2 \\ &= F(x_0 + 1, y_0 - 0, 5) + 2x_0 + 5 - 2y_0 \\ &= F(x_0 + 1, y_0 - 0, 5) + 2(x_0 - y_0) + 5 \end{aligned}$$

What's missing? The initialization contains a floating-point number.

Either multiply everything by 4

or initialize d with $1 - r$. Why that?

if $d + 1,25 - r > 0 \Rightarrow d + 1 - r > 0$ for integer values

if $d + 1,25 - r < 0 \Rightarrow d + 1 - r < 0$ for integer values