Exercise Computer graphics – (till March 17, 2009)

Ultra-fast line drawing

Exercise 6: In the lecture we have seen the mid-point algorithm for ellipses, however, only the part of the ellipse between 45° and 90° was drawn. Finish the arc between 0° and 45° and include it into the application.

Solution:

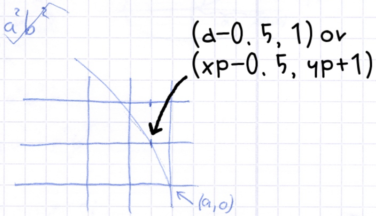
$F(a,0)=0$

$F(a-0{,}5,1) = b^2(a-0{,}5)^2 + a^2 1^2 - a^2b^2$

$= b^2(a^2-a+0{,}25) + a^2 - a^2b^2$

$= \cancel{b^2a^2} - b^2a + 0{,}25b^2 + a^2 - \cancel{a^2b^2}$

$= a^2 + b^2(0{,}25-a)$



$F(a,0)=0$

1. case $d > 0 \Rightarrow$ choose $N \Rightarrow y{+}{+};$

2nd region

$d\_new = F(x_0-0{,}5, y+2)$

$= b^2(x_0-0{,}5)^2 + a^2(y+2)^2 - a^2b^2$

$= b^2x_0^2 - b^2x_0 + b^2 \cdot 0{,}25 + a^2y^2 + a^2 \cdot 4 \cdot y + a^2 4 - a^2b^2$

$= F(x_0-0{,}5, y+1) + 2a^2y + 3a^2$

$F(x_0-0{,}5, y_0+1)$

$= b^2(x_0-0{,}5)^2 + a^2(y_0+1)^2 - a^2b^2$

$= b^2x_0^2 - b^2x_0 + 0{,}25b^2 + a^2y_0^2 + 2a^2y_0 + a^2 - a^2b^2$

2. case $d<0 \Rightarrow$ choose $NW \Rightarrow x{-}{-};\ y{+}{+};$

$d\_new = F(x_0-1{,}5; y+2)$

$= b^2(x_0-1{,}5)^2 + a^2(y+2)^2 - a^2b^2$

$= b^2x_0^2 - 3b^2x_0 + b^2 \cdot 2{,}25 + a^2y^2 + 4a^2y + a^2 4 - a^2b^2$

$= F(x_0-0{,}5; y+1) + (-2b^2x_0) + 2b^2 + 2a^2y + 3a^2$

$= F(x_0-0{,}5; y+1) + b^2(2-2x_0) + a^2(2y+3)$

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Exercise 7: Reduce the ellipse-version of the mid-point algorithms to circles.
Why is this version faster?

Solution:

Circle equation:

$$x^2 + y^2 = r^2$$

$$\Rightarrow F(x,y) = x^2 + y^2 - r^2$$

Initialization: $P_0 = (0, r)$

Pixel $(0, r)$ must lie on the circle

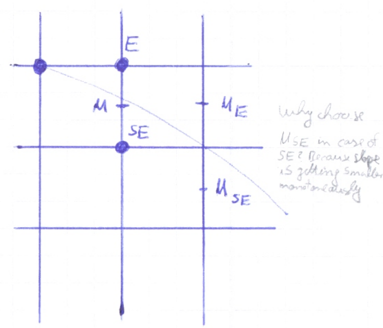
Thus $F(0, r) = 0^2 + r^2 - r^2 = 0$

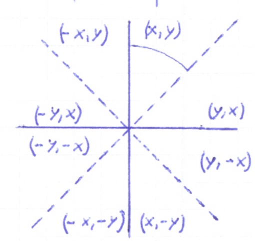
For the first M it holds true that $M = (1, r - \frac{1}{2})$

$$F(1, r - \tfrac{1}{2}) = 1^2 + (r - \tfrac{1}{2})^2 - r^2 =$$

$$1 + \cancel{r^2} - r + 0{,}25 - \cancel{r^2}$$

$$= 1{,}25 - r = d\_old$$





1. Fall: $d\_old < 0 \hat{=} M$ below circle so choose $E \Rightarrow x = x + 1$

$$M_E = (x_0 + 2, \; y_0 - 0{,}5)$$

$$d\_new = F(x_0 + 2, y_0 - 0{,}5) = (x_0 + 2)^2 + (y_0 - 0{,}5)^2 - r^2$$

$$= x_0^2 + 4x_0 + 4 + y_0^2 - y_0 + 0{,}25 - r^2$$

$$\boxed{\begin{aligned} &F(x_0 + 1, y_0 + 0{,}5) \\ &= (x_0+1)^2 + (y_0 - 0{,}5)^2 - r^2 \\ &= x_0^2 + 2x_0 + 1 + y_0^2 - y_0 + 0{,}25 - r^2 \end{aligned}}$$

$$= \underbrace{F(x_0 + 1, y_0 + 0{,}5)}_{d\_old} + 2x_0 + 3$$

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Solution: (continued)

2. case: d_old > 0 ≙ M above circle so choose SE ⇒ x = x + 1; y = y − 1

$$M_{SE} = (x_0 + 2, y_0 - 1{,}5)$$

$$d\_new = F(x_0+2, y_0-1{,}5) = (x_0+2)^2 + (y_0-1{,}5)^2 - r^2$$

$$= x_0^2 + 4x_0 + 4 + y_0^2 - 3y_0 + 2{,}25 - r^2$$

$$= F(x_0+1, y_0-0{,}5) + 2x_0 + 5 - 2y_0$$

$$= F(x_0+1, y_0-0{,}5) + 2(x_0-y_0) + 5$$

What's missing? The initialization contains a floating-point number.

Either multiply everything by 4

or initialize d with 1 − r. Why that?

if $d + 1{,}25 - r > 0 \Rightarrow d + 1 - r > 0$ for integer values

if $d + 1{,}25 - r < 0 \Rightarrow d + 1 - r < 0$ for integer values