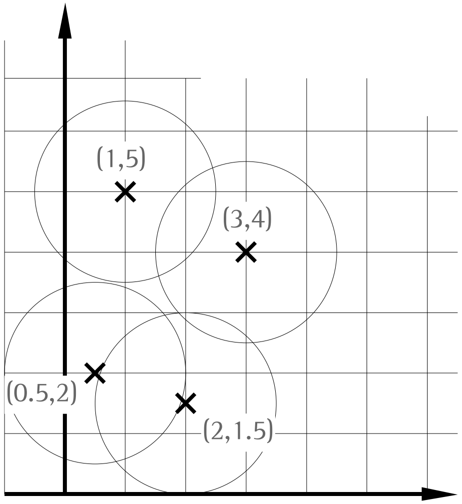
Exercise Sensor Networks - (till June 13, 2005)

Lecture 9: Localization in sensor networks

Exercise 9.1: Positioning without distance estimation

Nodes A and B do not know their positions but they can hear one another. Node A knows its neighbors (1,5) and (3,4) and B can hear its neighbors (0.5,2) and (2,1.5). The circular radios range of all nodes has a radios of 1.5 units.



a) Calculate whether (2,4) or (2,5) is a valid position for A.

b) In another setting only one node C is not positioned and a couple of neighbors exist with known positions. Again the distance between nodes can not be estimated but the position of C should be guessed (within its valid area). The radio range of all nodes is known.

i) What is the upper limit for the error between the estimated node position and its true position and and where do the positioned nodes have to be located so that the maximum error can occur?

ii) Can you image a configuration of nodes in which the error of the estimation can not become zero?

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Solution 9.1 a):

The solution works analog to the proceeding in the lecture slides “Localization – Simple global positioning”.

Node B has to be positioned within its valid area (lower gray patch in the Figure) so that it produces the largest possible area for A. In other words: More freedom for A is not possible. So B should be moved to position n.

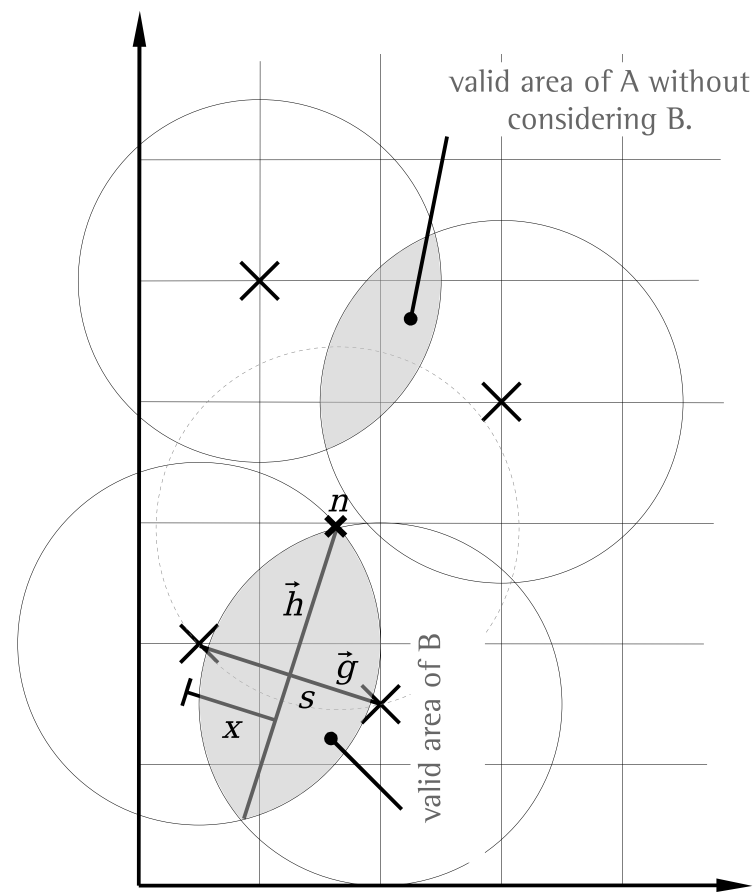
Here the calculation of n is easy since all circles have to same radius. That is why point s is in the middle of node B’s neighbors. The perpendicular onto the line between the neighbors already leads to n. The distance from s to n is determined by Pythagoras’ theorem.

$$s=\begin{pmatrix}0.5\\2\end{pmatrix}+0.5\left[\begin{pmatrix}2\\1.5\end{pmatrix}-\begin{pmatrix}0.5\\2\end{pmatrix}\right]=\begin{pmatrix}1.25\\1.75\end{pmatrix} \qquad h=\begin{pmatrix}0.5\\1.5\end{pmatrix}$$

$$1.5^2=d_{s,n}^2+\left(\frac{\sqrt{2.5}}{2}\right)^2 \Leftrightarrow d_{s,n}=\sqrt{1.625}$$

$$n=s+d_{s,n}\frac{1}{|h|}\vec{h} \qquad n=\begin{pmatrix}1.25\\1.75\end{pmatrix}+d_{s,n}\frac{1}{\sqrt{2.5}}\begin{pmatrix}0.5\\1.5\end{pmatrix}\approx\begin{pmatrix}1.65\\2.96\end{pmatrix}$$

Location (2,4) is in the range of n according the |(2,4)-n|=~1.1. The location (2,5) is beyond the range of n since |(2,5)-n|=~2.1. So (2,5) is no valid option for the position of A.



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Solution 9.1 b):

i)

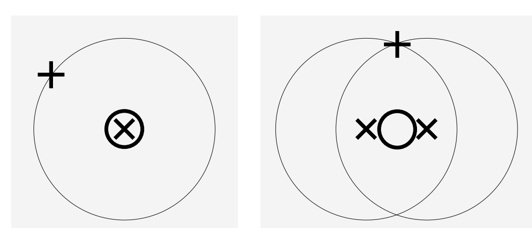
The upper bound for the error when guessing the position of a node can never be larger than the smallest radio range of all neighbors. The left sketch below shows a scenario with one neighbor only. In the worst case the non-positioned node is at the border of the range of its neighbor. If the positioning error was even greater the neighbor would not be known anymore.

Every additional node will never extend this error but it will possibly decrease the valid area of the non-positioned node like shown in the sketch below.

ii)

Of course the error between the assumed position and the true position can always be zero. If a node can hear its neighbor is can also (almost) be at its position.

+ true but unknown position of non-positioned node
× position of neighbor
○ Guessed position of non-positioned node



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Exercise 9.2: Relative localization by distance estimation

Node i was chosen as the center of the coordinate system of the whole sensor network. Another node k has also localized the neighbors of its own local view set within its own coordinate system. Now all nodes known to k but unknown to i should be added to node i's network-wide base.

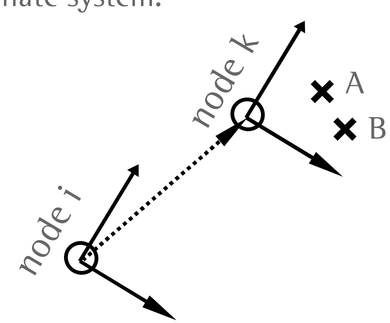
a) Why do the neighbors of k have to be transformed at all to fit into the global base? Wouldn't it have been much easier if k had chosen its y-axis in order to point upwards and the x-axis to point to the right?

Solution:

In the beginning each node defines its own base with itself as the center. At this point of time only the neighbors (its local view set) is known. The node has no idea about what is "above", "below", "left" or "right" (or north, south, etc. respectively). These can be defined arbitrarily. Later they will be chosen at random by the central node (denoted with i above) and all other nodes will have to adapt to this definition. Equal orientations of the local coordinate system could only be chosen if each node had e.g., a compass or some other global reference.

b) Draw a configuration of nodes in which the neighbors of a node k only have to be moved but not rotated in order to fit into the global coordinate system.

Solution:



Nodes A and B are defined with respect to node k. k's axes already point into the same direction as the axes of the global base of i so only a translation is necessary.

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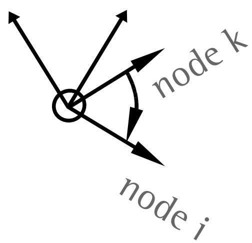
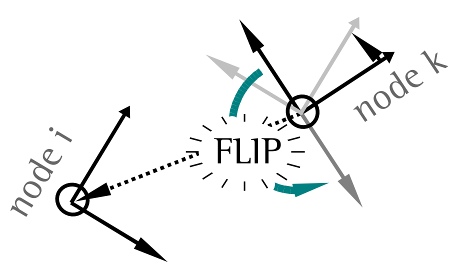
Solution 9.2 b):

c) Find a case in which k only has to be rotated but not translated. What is unusual about this case (is it realistic?)



If no translation is necessary node k and i have to be at the same place. This is theoretically not possible but could occur in reality due to imprecise distance estimates.

d) Find a case in which a translation, a rotation and mirroring of one axis is necessary.



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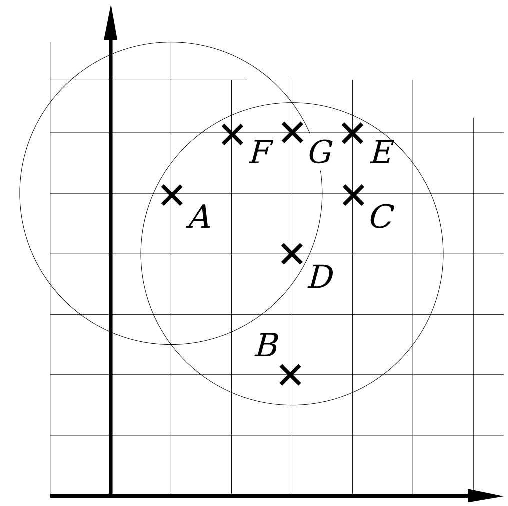
Exercise 9.3: Determination of local coordinates by distance estimates

Node A-G have different mutual distances which can be seen in the following table.

	A	B	C	D	E	F	G
A	0			2,24	3,16	1,41	2,24
B		0		2			
C			0	1,41	1	2,24	1,41
D	2,24	2	1,41	0	2,24	2,24	2
E	3,16		1	2,24	0	2	1
F	1,41		2,24	2,24	2	0	1
G	2,24		1,41	2	1	1	0



The sketch above is an example for a distribution of nodes which meet the constraints defined in the table of distances. Other configurations are possible as well.

No table entry means that no connection between nodes exist.

a) Which nodes are in particular suitable as origin of a global coordinate system and for defining the X-axis. Which nodes are least suitable and why?

Solution:

D is a good choice as origin because it can hear all other nodes resp., it has distance estimates to all other nodes. G or F could be used as x-axis because they are connected to all other nodes except for B. For the following solution we define:

D: origin
G: defines the x-axis
F: defines the direction of the y-axis

Node B is the least suitable node for defining the origin because D can only hear and estimate the distance to B. No other node could be localized with the help of node B.

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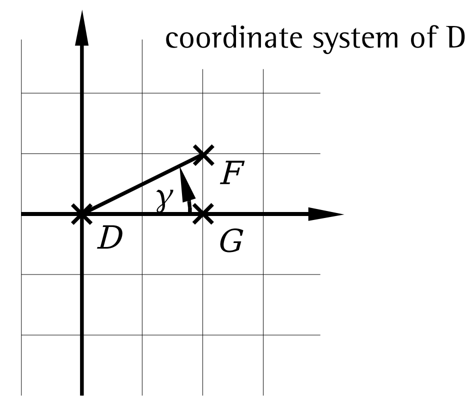
b) Node C has itself defined a local coordinate system and chose node E for defining the x-axis and node G for determining the direction of the y-axis. Determine the global [global as chosen in a)] coordinate of the local position (1,1) being defined from the viewpoint of C.

Solution:

The base (coordinate system) of D:



In the beginning we only know that D is the origin and the G defined the x-axis. Since the distance from D to G is 2 according to the table G's coordinates are (2,0). Now the coordinates of F has to be obtained. According to the law of cosines (see the lecture slides) the angle gamma between DG and DF is defined by:

$$\gamma = acos\left(\frac{2^2+2.24^2-1}{2\times2\times2.24}\right)\approx 26{,}6\,° \Rightarrow \quad F=2.24\times\begin{pmatrix}\cos(\gamma)\\ \sin(\gamma)\end{pmatrix}\approx\begin{pmatrix}2\\1\end{pmatrix}$$

The factor 2.24 for determining F is also taken from the table (distance DF). With the decision to chose sin(gamma) we have defined that F lies above the x-axis. Note that sin(-gamma)=(-1) would also have been a valid choice.

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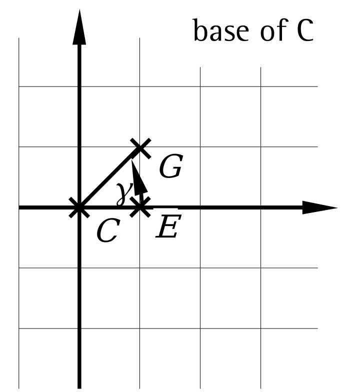
Exercise 9.3: Determination of local coordinates by distance estimates

Solution:

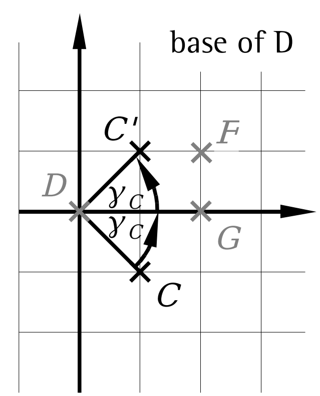
The base of C:



We have now determined the base of D and come to the one of C. In our solution C uses node E as x-axis and G as y-axis. We repeat the same kind of calculation as done for node D. The coordinate of E if give by E=(1,0) since the distance from C to E is 1 according to the table. For G we have to calculate:

$$\gamma = acos\left(\frac{1{,}41^2+1^2-1^2}{2\times 1{,}41\times 1}\right)\approx 45\,° \Rightarrow G = 1{,}41\times\begin{pmatrix}\cos(45\,°)\\ \sin(45\,°)\end{pmatrix}\approx\begin{pmatrix}1\\ 1\end{pmatrix}$$

Now the base of C has to be rotated to be aligned with the one of D. Obviously it has to be known how nodes C, E and G are localized with respect to D. Let's have a look at C first:



The angle between the x-axis of D and the line DC is:

$$\gamma_C = acos\left(\frac{1{,}41^2+1^2-1^2}{2\times 1{,}41\times 1^2}\right)\approx 45\,° \Rightarrow \qquad C = 1{,}41\times\begin{pmatrix}\cos(45\,°)\\ \pm\sin(45\,°)\end{pmatrix}\approx\begin{pmatrix}1\\ \pm 1\end{pmatrix}$$

Note that a case differentiation occurs here because the angle between D's x-axis and the line DC can either be drawn above or below the x-axis. The problem did not occur so far because we had the freedom to let the y-axis point into an arbitrary direction.

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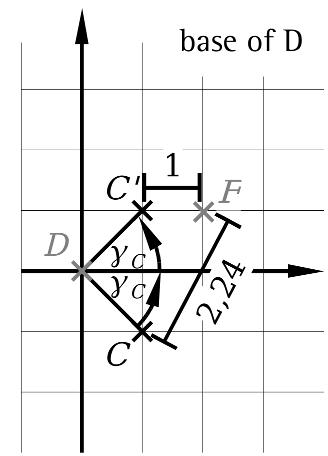
Exercise 9.3: Determination of local coordinates by distance estimates

Solution:

$$\gamma_C = acos\left(\frac{1{,}41^2+1^2-1^2}{2\times 1{,}41\times 1^2}\right)\approx 45\,°\Rightarrow \qquad C=1{,}41\times\begin{pmatrix}\cos(45\,°)\\ \pm\sin(45\,°)\end{pmatrix}\approx\begin{pmatrix}1\\ \pm 1\end{pmatrix}$$



Both cases (for C above or below) can be distinguished easily if the distance to node F is calculated. For (1,1) the distance is 1 however, according to the table it should be 2.24. This is only true for C=(1,-1).

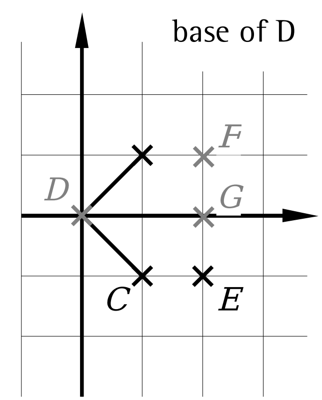
The same proceeding leads to a coordinate of node E within the base of D. G is already known from the previous calculation.

$$\gamma = acos\left(\frac{2^2+2{,}24^2-1}{2\times 2\times 2{,}24^2}\right)\approx 26{,}6\,°\Rightarrow \quad E=2{,}24\times\begin{pmatrix}\cos(\gamma)\\ \pm\sin(\gamma)\end{pmatrix}\approx\begin{pmatrix}2\\ \pm 1\end{pmatrix}$$

According to the table the distance EF should be 2. It follows that E=(2,-1).

At this point the task is almost done because fortunately the axes of D's base are parallel to the axes of C's base. So point (1,1) with respect to C only has to be transformed by the vector (1,-1) or in other words: The Base of C differs from the one of D only by this translation).

Solution for the transformed point: (2,0).



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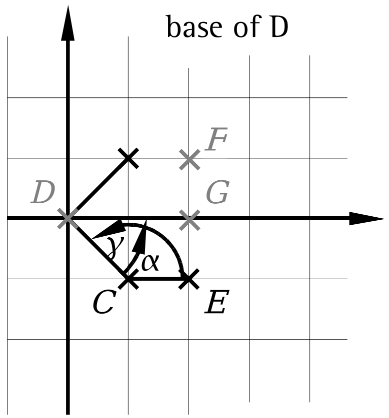
Solution:



If the axes of the coordinate system of C and D had not been parallel by chance an additional rotation would have been necessary.

$$\alpha = acos\left(\frac{1}{|E-C||D-C|}(E-C)\times(D-C)\right) = acos\left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}\right) = acos\left(\frac{-1}{\sqrt{2}}\right) = 135°$$

$$\gamma = 45°$$

In our particular case the rotation is not necessary since 135°+45°+180°=0°. For any other angle a rotation would have been necessary prior to the translation.

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Exercise 9.3: Determination of local coordinates by distance estimates

c) What are the problems when localizing node B? What is the candidate region for B? You only need to describe a proceeding, no actual coordinate.

Solution:

As node C has only one neighbor the valid area for its location can only be bounded to a certain degree. According to the table distance DB is known so B can only be situated on a circle around D with distance DB. At the same time we known that B can not be heard by A and C so these areas can be excluded from the circle. The white circle is the remainder in the sketch on the right side.