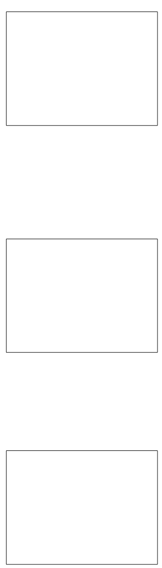
Exercise Computer graphics – (till November 20, 2007)

Rotations

Exercise 18: a) It is possible to decompose rotations into a number of succeeding shears. What is the least number of shears a rotation in 2D can be decomposed into? Explicitly state which shears you need.

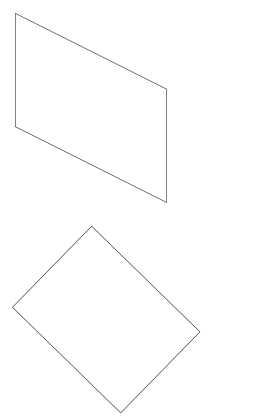
b) In which way does an image manipulation program benefit from the decomposition you suggested above?



horizontal shear =>

vertical shear =>

rotation =>



$$\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

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Solution b): The image can be rotated by shifting data within the memory only.

Express a 2D Rotation as 3 shears

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+st & s \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+st & u+stu+s \\ t & ut+1 \end{pmatrix}$$

I $\quad 1+st = \cos\alpha$

II $\quad u+s(tu+1) = -\sin\alpha$

III $\quad t = \sin\alpha \quad \Rightarrow \quad \underline{t=\sin\alpha}$

IV $\quad ut+1 = \cos\alpha$

III into I: $\quad 1+s\cdot\sin\alpha = \cos\alpha$

$$\underline{s = \frac{\cos\alpha - 1}{\sin\alpha}}$$

III into IV: $\quad u\cdot\sin\alpha + 1 = \cos\alpha$

$$\underline{u = \frac{\cos\alpha - 1}{\sin\alpha}}$$

check II:

$$\frac{\cos\alpha-1}{\sin\alpha} + \frac{\cos\alpha-1}{\sin\alpha}\left(\cancel{\sin\alpha}\,\frac{\cos\alpha-1}{\cancel{\sin\alpha}}+1\right) = -\sin\alpha \quad / \cdot \sin\alpha$$

$$\cancel{\cos\alpha} - 1 + \cos^2\alpha - \cancel{\cos\alpha} = -\sin^2\alpha$$

$$\cos^2\alpha - 1 = -\sin^2\alpha$$

$$\cos^2\alpha + \sin^2\alpha = 1 \quad \checkmark$$