

Exercise Computer graphics – (till November 13, 2007)

Bicubic/Bilinear surfaces

Exercise 17: We have seen how bi-cubic patches and planar patches of 1st degree were defined in the lecture.

- a) Define a patch of 2nd degree. The tangent on one side of the patch should be definable explicitly.

Solution:

SOLUTION EXERCISE

$$Q(t) = at^2 + bt + c$$

$$Q'(t) = 2at + b$$

① $Q(0) = P_1(s) = c$
 ② $Q_s(1) = P_2(s) = a + b + c$
 ③ $Q'_t(0) = R(s) = b$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} P_1(s) \\ P_2(s) \\ R(s) \end{pmatrix}$$

So the polynomial which satisfies constraint ①, ② and ③ is def. as:

$$\begin{pmatrix} t^2 & t & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_1(s) \\ P_2(s) \\ R(s) \end{pmatrix} = Q(t)$$

Now, the curves $P_1(s)$, $P_2(s)$ and $R(s)$ have to be defined as well

$$P_1(s) = (s^2 + s - 1) \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_{11} \\ k_{12} \\ k_{11} - k_{10} \end{pmatrix}; \quad P_2(s) = (s^2 + s - 1) \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_{21} \\ k_{22} \\ k_{21} - k_{20} \end{pmatrix}$$

$$R(s) = (s^2 + s - 1) \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_{01} \\ k_{02} \\ (k_{11} - k_{10}) - (k_{01} - k_{00}) = z \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} P_1(s) & P_2(s) & R(s) \end{pmatrix} = (s^2 + s - 1) \begin{pmatrix} -1 & 1 & -1 & k_{11} & k_{21} & k_{01} \\ 0 & 0 & 1 & k_{12} & k_{22} & k_{02} \\ 1 & 0 & 0 & k_{11} - k_{10} & k_{21} - k_{20} & z \end{pmatrix}$$

$$\Rightarrow Q(s, t) = \begin{pmatrix} t^2 & t & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_{11} & k_{12} & k_{11} - k_{10} \\ k_{21} & k_{22} & k_{21} - k_{20} \\ k_{01} & k_{02} & z \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 0 \end{pmatrix} (s^2 + s - 1)$$

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Exercise 17: We have seen how bi-cubic patches and planar patches of 1st degree were defined in the lecture.

- b) Interpret the influence of all components of the geometry matrix on the patch.

Interpretation of the geometry matrix:

$$\begin{pmatrix} Q(s=0; t=0)=k_{11} & Q(s=0; t=1)=k_{21} & \frac{Q(s=0; t=1) - Q(s=0; t=0)}{\partial t} \\ Q(s=1; t=0)=k_{12} & Q(s=1; t=1)=k_{22} & \frac{Q(s=1; t=1) - Q(s=1; t=0)}{\partial t} \\ \frac{Q(s=1; t=0) - Q(s=0; t=0)}{\partial s} & \frac{Q(s=1; t=1) - Q(s=0; t=1)}{\partial s} & \frac{Q(s=1; t=1) - Q(s=0; t=1) - Q(s=1; t=0) + Q(s=0; t=0)}{\partial s \partial t} \end{pmatrix}$$