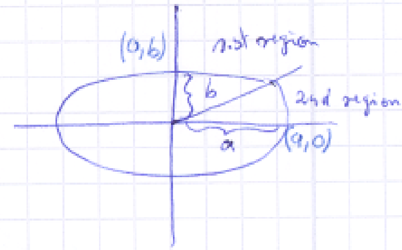
Scan-converting ellipses:

$F(x,y) = b^2x^2 + a^2y^2 - a^2b^2 = 0$



$\left[F(0,b) = b^2 0^2 + a^2b^2 - a^2b^2 = 0 \checkmark\right]$

d_old $F(1, b-0{,}5) = b^2 1^2 + a^2(b-0{,}5)^2 - a^2b^2$

$= b^2 + a^2(b^2 - b + 0{,}25) - a^2b^2 =$

$= b^2 + \cancel{a^2b^2} - a^2b + 0{,}25a^2 - \cancel{a^2b^2}$

1st region

1. case d > 0 ⇒ choose SE ⇒ x++; y--;

d_new $= F(x_0+2;\ y_0-1{,}5) =$

$b^2(x_0+2)^2 + a^2(y_0-1{,}5)^2 - a^2b^2$

$= b^2(x_0^2 + 4x_0 + 4) + a^2(y_0^2 - 3y_0 + 2{,}25) - a^2b^2$

$= b^2x_0^2 + 4b^2x_0 + 4b^2 + a^2y_0^2 - 3a^2y_0 + 2{,}25a^2 - a^2b^2$

$= F(x_0+1, y_0-0{,}5) + 2b^2x_0 + 3b^2 - 2a^2y_0 + 2a^2$

$= F(x_0+1, y_0-0{,}5) + b^2(2x_0+3) + a^2(2-2y_0)$

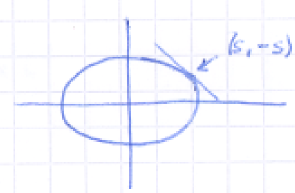
$$\left[\begin{aligned} &F(x_0+1,\ y_0-0{,}5) = \\ &= b^2(x_0+1)^2 + a^2(y_0-0{,}5)^2 - a^2b^2 \\ &= b^2(x_0^2+2x_0+1) + a^2(y_0^2 - y_0 + 0{,}25) - a^2b^2 \\ &= b^2x_0^2 + 2b^2x_0 + b^2 + a^2y_0^2 - a^2y_0 + 0{,}25a^2 - a^2b^2 \end{aligned}\right]$$

2. case d < 0 ⇒ choose E ⇒ x++;

d_new $= F(x_0+2,\ y_0-0{,}5) =$

$F(x_0+1, y_0-0{,}5) + b^2(2x_0+3)$

Where is (s,-s)?



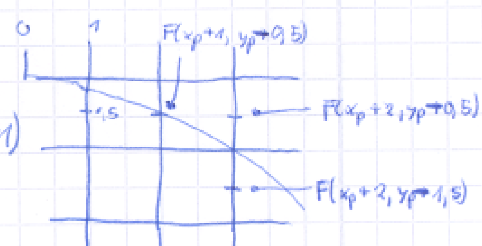
$\dfrac{\partial F(x,y)}{\partial x} = 2b^2x$

For a new/next mid-point it has to hold true that:

$\dfrac{\partial F(x,y)}{\partial y} = 2a^2y$

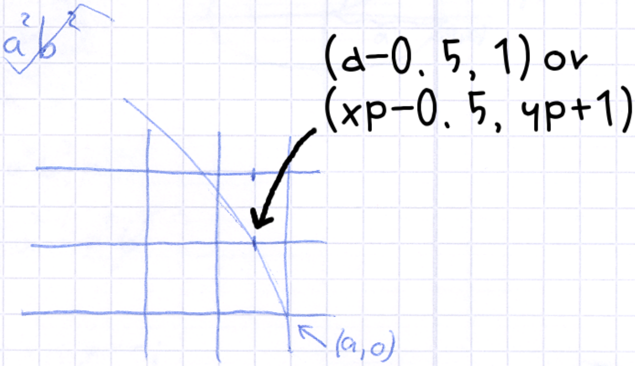
$2a^2(y-1) > 2b^2(x+1)$

So iterate x from X=0 until $a^2(y-1) < b^2(x+1)$



$F(a,0)=0$

$$F(a-0{,}5,1) = b^2(a-0{,}5)^2 + a^2 1^2 - a^2b^2$$
$$= b^2(a^2-a+0{,}25) + a^2 - a^2b^2$$
$$= \cancel{b^2a^2} - b^2a + 0{,}25b^2 + a^2 - \cancel{a^2b^2}$$
$$= a^2 + b^2(0{,}25-a)$$



$F(a,0)=0$

1. case $d > 0$ ⇒ choose N ⇒ y++;

2nd region

$$d\_new = F(x_0-0{,}5, y+2)$$
$$= b^2(x_0-0{,}5)^2 + a^2(y+2)^2 - a^2b^2$$
$$= b^2x_0^2 - b^2x_0 + b^2\cdot 0{,}25 + a^2y^2 + a^2\cdot 4\cdot y + a^2 4 - a^2b^2$$
$$= F(x_0-0{,}5, y+1) + 2a^2y + 3a^2$$

$$F(x_0-0{,}5, y_0+1)$$
$$= b^2(x_0-0{,}5)^2 + a^2(y_0+1)^2 - a^2b^2$$
$$= b^2x_0^2 - b^2x_0 + 0{,}25b^2 + a^2y_0^2 + 2a^2y_0 + a^2 - a^2b^2$$

2. case $d<0$ ⇒ choose NW ⇒ x−−; y++;

$$d\_new = F(x_0-1{,}5; y+2)$$
$$= b^2(x_0-1{,}5)^2 + a^2(y+2)^2 - a^2b^2$$
$$= b^2x_0^2 - 3b^2x_0 + b^2\cdot 2{,}25 + a^2y^2 + 4a^2y + a^2 4 - a^2b^2$$
$$= F(x_0-0{,}5; y+1) + (-2b^2x_0) + 2b^2 + 2a^2y + 3a^2$$
$$= F(x_0-0{,}5; y+1) + b^2(2-2x_0) + a^2(2y+3)$$