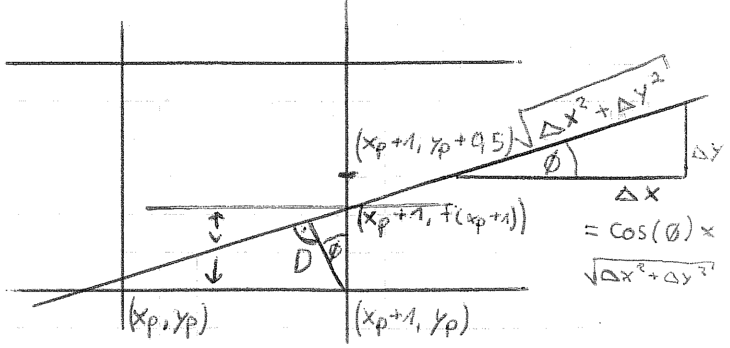
CASE FOR CHOOSING E

10

$$\underbrace{D}_{\Delta x} = V\cos(\phi) \quad \text{known}$$

$$\Delta X = \underbrace{\cos(\phi)}_{\text{needed}}\sqrt{\Delta x^2 + \Delta y^2}$$

$$\Rightarrow \cos(\phi) = \frac{\Delta X}{\sqrt{\Delta x^2 + \Delta y^2}}$$



$$D = \frac{V\Delta X}{\sqrt{\Delta x^2 + \Delta y^2}}$$

We did already obtain $F(x_p+1, y_p+0{,}5)$

$$F(x_p+1, y_p+0{,}5) = \Delta y(x_p+1) - \Delta x(y_p+0{,}5) + \Delta x b$$

$$= \Delta y x_p + \Delta y - \Delta x y_p - \Delta x \cdot 0{,}5 + \Delta x b$$

Why not calculate $F(x_p+1, y_0) \equiv D$ directly?

1. Δx and Δy integer $\Rightarrow D$ not real distance
2. We want to base our calculation on the distance $F(x_p+1, y_p+0{,}5)$ which is already available

$$V = f(x_p+1) - (y_p + 1 - 1)$$

$$\Leftrightarrow V = \frac{\Delta y}{\Delta x}(x_p+1) + b - y_p \underset{+1}{-1} \cdot \Delta x$$

in addition for pixel NE

$$y = \frac{\Delta y}{\Delta x} x + b \quad / \cdot \Delta x$$

$$\Delta x y = \Delta y x + \Delta x b$$

$$0 = x\Delta y - y\Delta x + b\Delta x = f(x,y)$$

$$V\Delta X = \Delta y(x_p+1) + b\Delta x - y_p\Delta x \;\underset{+\Delta x}{-\Delta x}$$

$$= \Delta y x_p + \Delta y + b\Delta x - y_p\Delta x \;\underset{+\Delta x}{-\Delta x}$$

$$= F(x_p+1, y_p+0{,}5) + 0{,}5\Delta x \;\underset{+\Delta x}{-\Delta x}$$

$\Rightarrow$ The pixel color is based on the distance $\dfrac{d\_old + 0{,}5\Delta X}{\sqrt{\Delta x^2 + \Delta y^2}} = \overbrace{\dfrac{2d\_old + \Delta X}{2\sqrt{\Delta x^2 + \Delta y^2}}}^{\cdot 2 \text{ in order to preserve integer arithmetic}}$

Pixel NE: $\dfrac{d\_old + 0{,}5\Delta x - \Delta x}{\sqrt{x^2 + y^2}}$

Hint: Depending on your calculation of d_old, it might already include the factor 2!

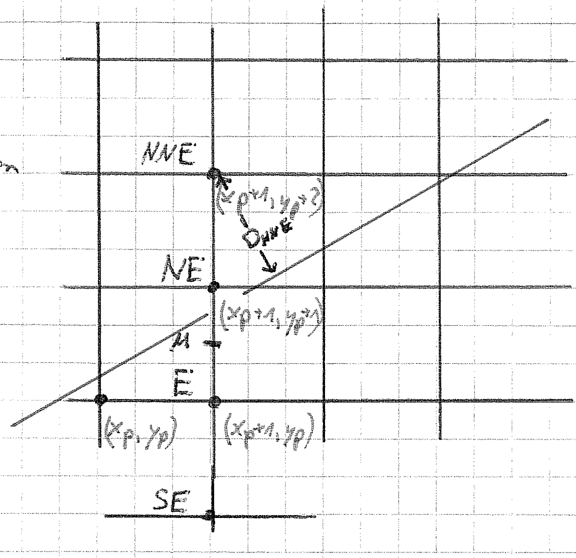
Pixel SE: $\dfrac{d\_old + 0{,}5\Delta x + \Delta x}{\sqrt{x^2 + y^2}}$

$|max\_dist| = 1{,}5$

scale $0, \dots, 1{,}5$ as $255, \dots, 0$

CASE FOR CHOOSING NE

Here, we choose pixel NE. Again, the pixel above and below has to be 'colored'. We did the computation for (x_p+1, y_p) and (x_p+1, y_p+1) already. So now, calculate the distance line $\longleftrightarrow (x_p+1, y_p+2)$



$$D = \frac{v \cdot \Delta x}{\sqrt{\Delta x^2 + \Delta y^2}}$$

Hint: Note that v may be negative which is also necessary in order to calculate the correct distances

$$v = f(x_p+1) - (y_p+2)$$

$$\Longleftrightarrow v = \frac{\Delta y}{\Delta x}(x_p+1) + b - (y_p+2) \quad / \cdot \Delta x$$

$$\vdots$$

$$\underline{\underline{v \Delta x = F(x_p+1, y_p+0{,}5) - 1{,}5 \Delta x}}$$

Summary:

$$D_{SE} = (d_{old} + 1{,}5 \Delta x) / \sqrt{\Delta x^2 + \Delta y^2}$$

$$D_{E} = (d_{old} + 0{,}5 \Delta x) / \sqrt{\Delta x^2 + \Delta y^2}$$

$$D_{NE} = (d_{old} - 0{,}5 \Delta x) / \sqrt{\Delta x^2 + \Delta y^2}$$

$$D_{NNE} = (d_{old} - 1{,}5 \Delta x) / \sqrt{\Delta x^2 + \Delta y^2}$$