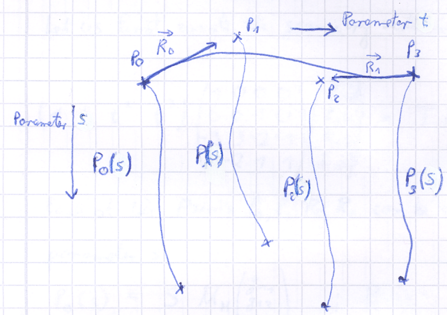
Computer graphics

Bicubic surfaces

Parametric Bicubic Surfaces (1)



Initially our curve has the following form:

$\cancel{a + bx + cx^2 + dx^3 =}$

$$a + bt + ct^2 + dt^3 = Q(t) \;;\; b + 2cx + 3dx^2 = Q'(t)$$

The coefficients had to meet the following constraints:

$$\begin{aligned} Q(0) &= P_0 \Rightarrow a = P_0 \\ Q(1) &= P_1 \Rightarrow a+b+c+d = P_1 \\ Q'(0) &= R_0 \Rightarrow b = R_0 \\ Q'(1) &= R_1 \Rightarrow b+2c+3d = R_1 \end{aligned} \Rightarrow \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} P_0 \\ P_1 \\ R_0 \\ R_1 \end{pmatrix}$$

Now we bring the coefficants to the other side:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} P_0 \\ P_1 \\ R_0 \\ R_1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ R_0 \\ R_1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

The curve is defined as follows:

$$\begin{pmatrix} 1 & t & t^2 & t^3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ R_0 \\ R_1 \end{pmatrix} = Q(t)$$

$$\begin{pmatrix} 1-3t^2+2t^3 & 3t^2-2t^3 & t-2t^2+t^3 & -t^2+t^3 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ R_0 \\ R_1 \end{pmatrix} =$$

$$P_0(1-3t^2+2t^3) + P_1(3t^2-2t^3) + R_0(t-2t^2+t^3) + R_1(-t^2+t^3) = Q(t)$$

Computer graphics

Bicubic surfaces

Parametric Bicubic Surfaces (2)

We now extent the curve to a surface:

Let us first write the geometry vector in parametric form:

$$(1\ \ t\ \ t^2\ \ t^3)\begin{pmatrix}1&0&0&0\\0&0&1&0\\-3&3&-2&-1\\2&-2&1&1\end{pmatrix}\begin{pmatrix}P_0(s)\\P_1(s)\\R_0(s)\\R_1(s)\end{pmatrix} = T\,M_H\,G_H(s)$$

Each of the Points P_0, P_1, R_0, R_1 can be written as a curve with parameter s.

$$P_0(s) = S\,M_H\begin{pmatrix}g_{11}\\g_{12}\\g_{13}\\g_{14}\end{pmatrix} \quad ; \quad P_1(s) = S\,M_H\begin{pmatrix}g_{21}\\g_{22}\\g_{23}\\g_{24}\end{pmatrix}$$

$$R_0(s) = S\,M_H\begin{pmatrix}g_{31}\\g_{32}\\g_{33}\\g_{34}\end{pmatrix} \quad ; \quad R_1(s) = S\,M_H\begin{pmatrix}g_{41}\\g_{42}\\g_{43}\\g_{44}\end{pmatrix}$$

Obviously we can write the "parametric" Points in the following form:

$$\left(P_0(s)\ \ P_1(s)\ \ R_0(s)\ \ R_1(s)\right) = S\,M_H\begin{pmatrix}g_{11}&g_{21}&g_{31}&g_{41}\\g_{12}&g_{22}&g_{32}&g_{42}\\g_{13}&g_{23}&g_{33}&g_{43}\\g_{14}&g_{24}&g_{34}&g_{44}\end{pmatrix}$$

In order to be used as Geometry vector we need the four points written in column-format which is done using the following equality:

$$(A\ B\ C)^T = C^T B^T A^T$$

Thus

$$\begin{pmatrix}P_0(s)\\P_1(s)\\R_0(s)\\R_1(s)\end{pmatrix} = G^T M_H^T S^T$$

yielding

$$Q(t,s) = T\,M_H\,G^T M_H^T S^T$$

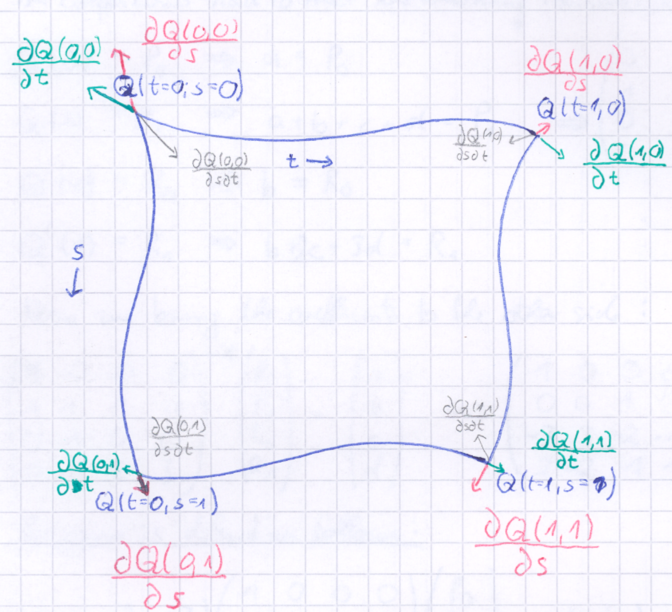
Computer graphics

Bicubic surfaces

Let's have a look at the meaning of the elements of the geometry matrix

~~Q(t=0;s=0) ∂Q(t=0,s=0)/∂s Q(~~

$$
\begin{pmatrix}
Q(t=0;s=0) & Q(t=0;s=1) & \frac{\partial Q(t=0,s=0)}{\partial s} & \frac{\partial Q(t=0;s=1)}{\partial s} \\
Q(t=1;s=0) & Q(t=1;s=1) & \frac{\partial Q(t=1;s=0)}{\partial s} & \frac{\partial Q(t=1,s=1)}{\partial s} \\
\frac{\partial Q(t=0,s=0)}{\partial t} & \frac{\partial Q(t=0,s=1)}{\partial t} & \frac{\partial Q(t=0;s=0)}{\partial s \partial t} & \frac{\partial Q(t=0;s=1)}{\partial s \partial t} \\
\frac{\partial Q(t=1;s=0)}{\partial t} & \frac{\partial Q(t=1;s=1)}{\partial t} & \frac{\partial Q(t=1;s=0)}{\partial s \partial t} & \frac{\partial Q(t=1,s=1)}{\partial s \partial t}
\end{pmatrix}
$$



Bicubic Surfaces (3)