Lecture on Sensor Networks

Timing and applications

Localization by Time Difference Of Arrival (TDOA)

Localizing events can be done based on precise time stamps if they are delivered along with measurements and if nodes are positioned.

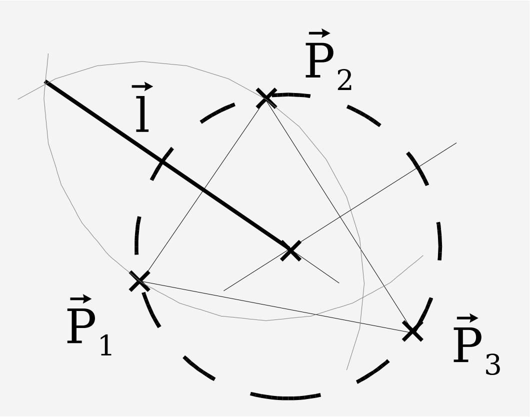
Example:

Nodes 1-3 are located at P_1,P_2,P_3. They encounter the same event at times t_1, t_2, t_3 respectively. Sound travels at 300m per second, with the same speed into all directions. All nodes lie approximately on the same plane.

Case 1: $t_1=t_2=t_3$. Where is the object?

Given $P_1=(-2,2)$ / $P_2=(1,1)$ / $P_3=(-1,-2)$

Geometric solution: Draw the perpendicular l through the middle of the line between P_1 and P_2. All points on l have equal distance to P_1 and P_2. From now on only those points are in question. We could, e.g., choose the one whose radius intersects P_1, P_2 and P_3 however, this is not easy to construct. Alternatively, a second perpendicular can be used like shown in the sketch on the right.



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Computational solution:

$$\vec{r}=\vec{P}_2-\vec{P}_1=\begin{pmatrix}+3\\-1\end{pmatrix} \qquad \vec{M}=\vec{P}_1+0.5\,\vec{r}=\begin{pmatrix}-0,5\\+1,5\end{pmatrix}$$

in 2D the following holds true:

$$\vec{l}=\begin{pmatrix}+r_y\\-r_x\end{pmatrix} \qquad \text{Check:} \qquad \vec{l}\,\vec{r}=\begin{pmatrix}+r_y\\-r_x\end{pmatrix}\begin{pmatrix}r_x\\r_y\end{pmatrix}=r_y r_x-r_x r_y=0$$

in particular: $\vec{l}=\begin{pmatrix}-1\\-3\end{pmatrix}$

All points x are addressed by parameter p:

$$\vec{x}_p=\vec{M}+p\,\vec{l}=\begin{pmatrix}-0,5\\+1,5\end{pmatrix}+p\begin{pmatrix}-1\\-3\end{pmatrix}$$

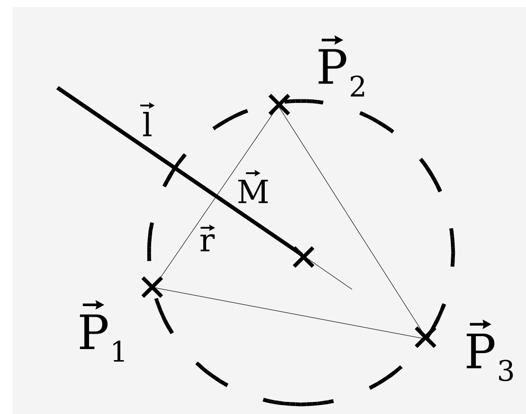
Find the p with the following properties:

$$|x_p-P_1|^2=|x_p-P_3|^2$$

$$\left[\begin{pmatrix}-0,5\\+1,5\end{pmatrix}+p\begin{pmatrix}-1\\-3\end{pmatrix}-\begin{pmatrix}-2\\+2\end{pmatrix}\right]^2=\left[\begin{pmatrix}-0,5\\+1,5\end{pmatrix}+p\begin{pmatrix}-1\\-3\end{pmatrix}-\begin{pmatrix}-1\\-2\end{pmatrix}\right]^2$$

$$18,5-2p8+10p^2=2,5+2p3+10p^2$$

$p=\frac{15}{24}$ final solution for the position: $\begin{pmatrix}-0,5\\+1,5\end{pmatrix}+\frac{15}{24}\begin{pmatrix}-1\\-3\end{pmatrix}=\begin{pmatrix}-1,125\\-0,375\end{pmatrix}$



Hint: The approach works analog for the case $t_1=t_2=(t_3+\text{delta})$ you will find in the exercise.

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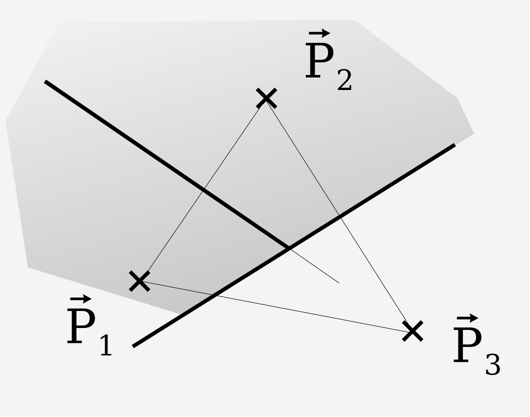
As we have seen in the lecture on time synchronization we can not always obtain precise time stamps. Sometimes only relationships between time stamps are available.

Case 2: $t_1=t_2$, $t_3 > t_2$. To what degree can the event's position be estimated?

Because of $t_1=t_2$ the source still has to lie on the perpendicular between P_1 and P_2.

The perpendicular between P_2 and P_3 divides the space into two half-spaces. Since P_3 encountered the event later than P_2, an additional constraint for the event is that it can only be positioned in the half-space which is further away from P_3.

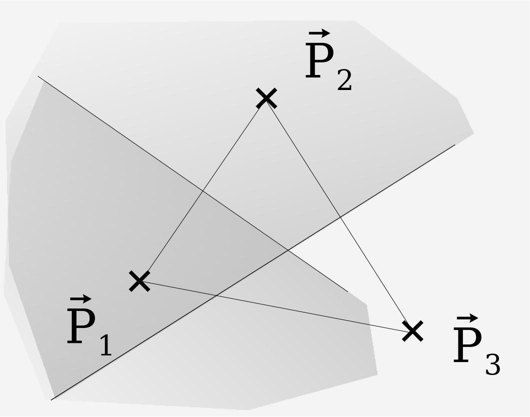


Timing and applications

Localization by Time Difference Of Arrival (TDOA)

Case 3: $t_1 < t_2 < t_3$. What is the valid area for the event?

Again, the perpendiculars divide the space into two half-spaces each. Only the intersecting area can contain the event. In the right sketch the event's distance to P_2 must be larger than the one to P_1 and at the same time P_1 must be nearer than P_3 which only leaves the darkest intersecting area.



How can an object be positioned if precise time differences are available?

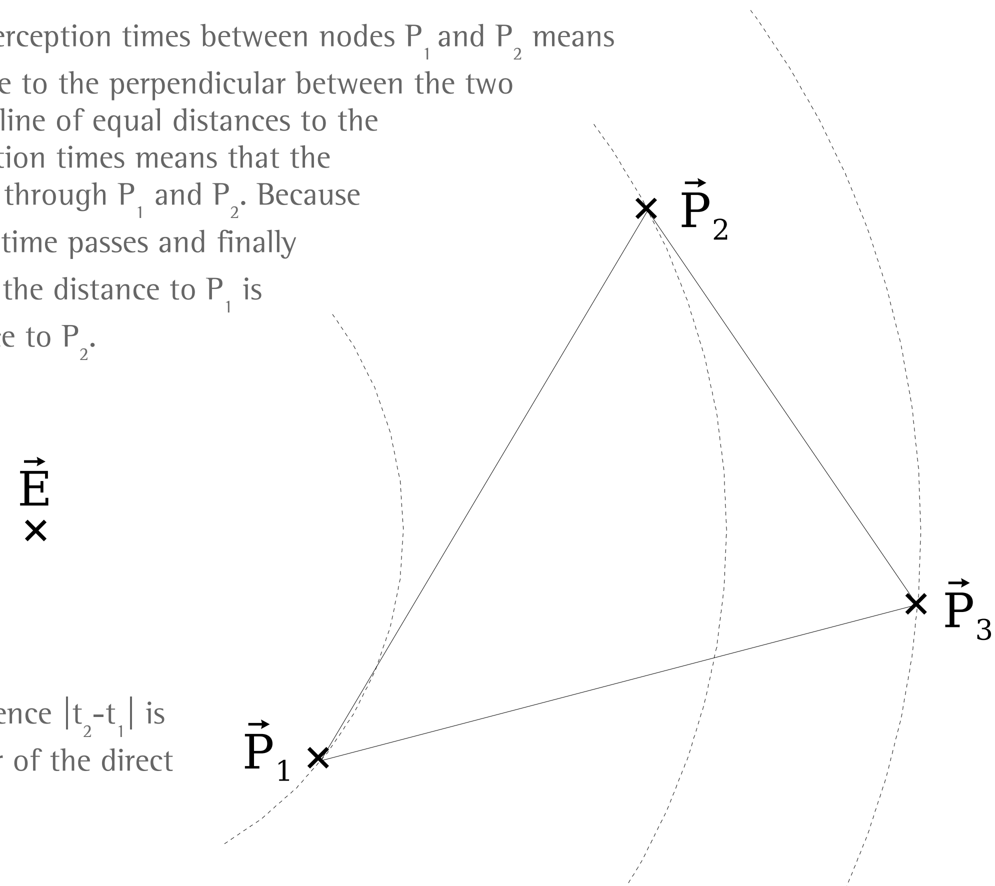
Timing and applications

Localization by Time Difference Of Arrival (TDOA)

Case 4: The precise times t_1, t_2 and t_3 the same event was encountered by all three nodes is known now. In this case the position of an event E can be estimated. With only two nodes only a rough approximation of the direction of the incoming signal can be guessed. Without loss of generality we assume $t_1<t_2<t_3$

Observation: A small difference of perception times between nodes P_1 and P_2 means that the event has to be located close to the perpendicular between the two nodes since the perpendicular is the line of equal distances to the nodes. An large difference of perception times means that the event is converging towards the line through P_1 and P_2. Because it first arrives at node P_1 ,then some time passes and finally P_2 is reached. This is only possible if the distance to P_1 is significantly smaller than the distance to P_2.

The time difference $|t_2-t_1|$ between the nodes can at most become $|P_1-P_2|/300$ seconds if sound travels at 300m/s. If the maximum time difference is measured the event must lie on a line through the nodes.



If on the other hand, the time difference $|t_2-t_1|$ is zero then E lies on the perpendicular of the direct line between the nodes.

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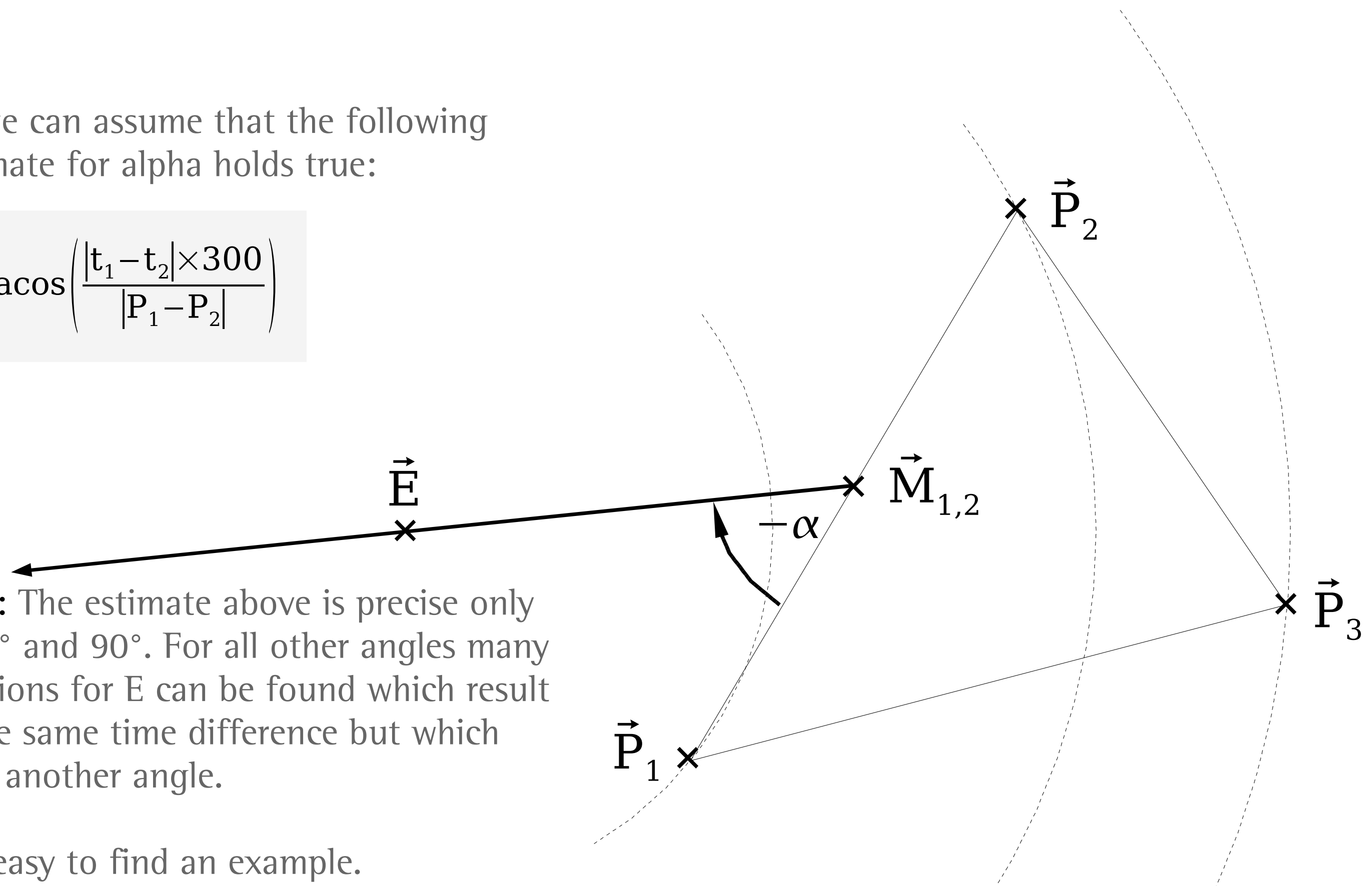
Localization by Time Difference Of Arrival (TDOA)

M shall be the middle of the line between P_1 and P_2. If the time difference $|t_1 - t_2|$ is zero the angle alpha between the line through P1, P2 and the line through E, M is 90°. If the time difference is at its maximum, the angle is 0°.

So we can assume that the following estimate for alpha holds true:

$$\alpha \approx \mathrm{acos}\left(\frac{|t_1 - t_2| \times 300}{|P_1 - P_2|}\right)$$



Note: The estimate above is precise only for 0° and 90°. For all other angles many locations for E can be found which result in the same time difference but which yield another angle.

It is easy to find an example.

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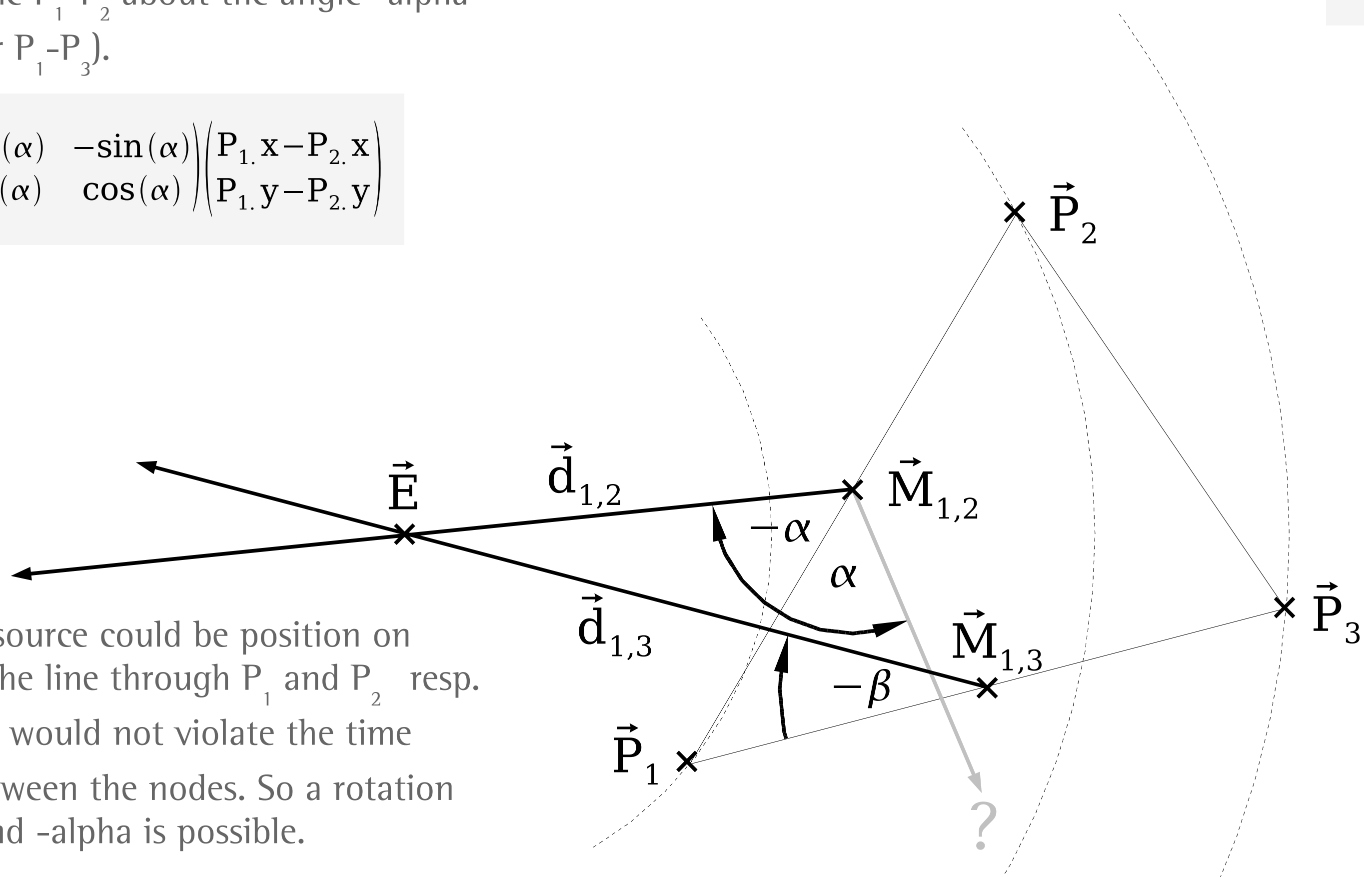
With the estimate for two angles, e.g. for the pair of nodes P_1,P_2 and P_1, P_3 the two evolving lines can be intersected like shown below in order to determine the position of E.

First, the vector d has to be computed by rotating the line P_1-P_2 about the angle -alpha (resp. -beta for P_1-P_3).

$$\begin{pmatrix} d_{1,2}.x \\ d_{1,2}.y \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} P_{1.}x - P_{2.}x \\ P_{1.}y - P_{2.}y \end{pmatrix}$$



Note that the source could be position on either side of the line through P_1 and P_2 resp. P_1 and P_3. This would not violate the time differences between the nodes. So a rotation about alpha and -alpha is possible.

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Now line 1 though $M_{1,2}$ and direction $d_{1,2}$ has to be intersected with line 2 through $M_{1,3}$ and direction $d_{1,3}$. The intersection gives a hint were event E might be positioned.

$x_I = M_{1,2} + r \times d_{1,2}$ Points on line 1

$x_{II} = M_{1,3} + s \times d_{1,3}$ Points on line 2

$M_{1,3} + s \times d_{1,3} = M_{1,2} + r \times d_{1,2} \Leftrightarrow B = r \times d_{1,2} - s \times d_{1,3}$ substituting $B = M_{1,3} - M_{1,2}$

$$\begin{pmatrix} B.x \\ B.y \end{pmatrix} = r \times \begin{pmatrix} d_{1,2}.x \\ d_{1,2}.y \end{pmatrix} - s \times \begin{pmatrix} d_{1,3}.x \\ d_{1,3}.y \end{pmatrix}$$

$r = \dfrac{B.x + s \times d_{1,3}.x}{d_{1,2}.x}$ $s = \dfrac{r \times d_{1,2}.y - B.y}{d_{1,3}.y}$ insert s in expression for r

$$r \times d_{1,2}.x \times d_{1,3}.y = B.x \times d_{1,3}.y + r \times d_{1,3}.x \times d_{1,2}.y - d_{1,3}.x \times B.y$$

$r \times (d_{1,2}.x \times d_{1,3}.y - d_{1,3}.x \times d_{1,2}.y) = B.x \times d_{1,3}.y - d_{1,3}.x \times B.y$ ...solve for variable r and substitute back

$$r = \frac{(M_{1,3}.x - M_{1,2}.x) d_{1,3}.y - (M_{1,3}.y - M_{1,2}.y) d_{1,3}.x}{d_{1,2}.x \times d_{1,3}.y - d_{1,3}.x \times d_{1,2}.y}$$

r is the parameter of line-equation 1 which leads to the assumed location of E.
Valid positions only evolve for r > 0 and s > 0!

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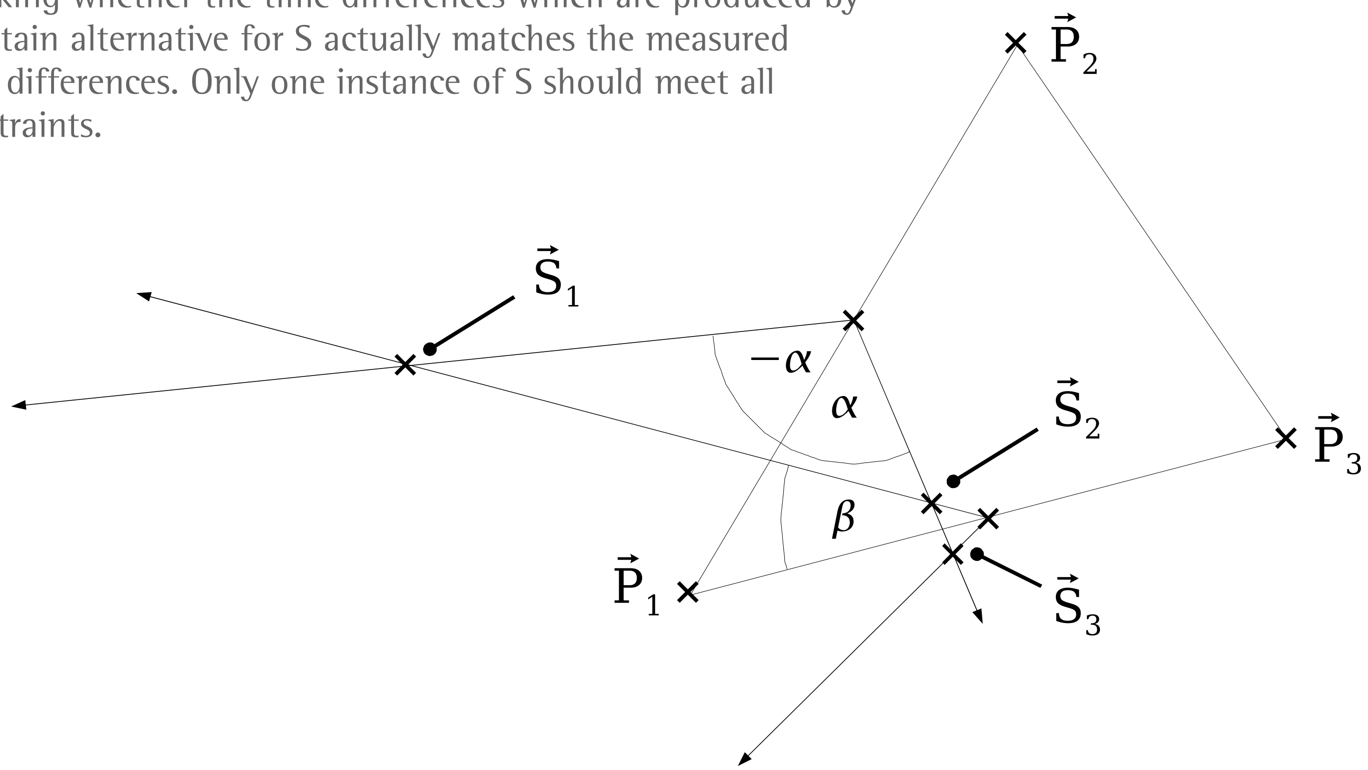
Each angle can be rotated clockwise or counter clockwise (alpha and beta in the sketch below) without getting into conflict with a constraint. The evolving three cases can be checked further using the remaining pair of nodes P_2 and P_3.

This can either be done by calculating a third direction or by checking whether the time differences which are produced by a certain alternative for S actually matches the measured time differences. Only one instance of S should meet all constraints.



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