Lecture on Sensor Networks

Communication in sensor networks

Cyclic Redundancy Check
CRC analysis
Aloha for sensor networks
Energy efficiency of Aloha
Simulation

Error control: Cyclic Redundancy Check (CRC)

CRC is based on the idea of polynomial division. Remember:

$$\begin{array}{l}
(x^5 + x^3 + x + 1) : (x + 1) = x^4-x^3+2x^2-2x+3-2/(x+1)\\
-(x^5 + x^4)\\
\hline
0 \quad - x^4 + x^3\\
\quad -(-x^4 - x^3)\\
\hline
\qquad 0+ 2x^3 + \qquad + x\\
\qquad -(2x^3 + 2x^2)\\
\hline
\qquad\quad 0 - 2x^2 \quad + \quad x\\
\qquad\quad -(-2x^2 \quad - 2x)\\
\hline
\qquad\qquad 0 \quad 3x + 1\\
\qquad\qquad -(3x + 3)\\
\hline
\qquad\qquad\quad 0 - 2 = \text{Remainder or modulus}
\end{array}$$

Check:
$[x^4-x^3+2x^2-2x+3-2/(x+1)]*(x+1)=...$

What's the difference between polynomial division and normal division?

-> There is no carry-over. Coefficients of a term x^n are only combined with coefficients of another term x^n.

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Error control: Cyclic Redundancy Check

A bit string is interpreted as a polynomial by numbering the bits consecutively and, if a bit is set, by adding the corresponding term of the polynomial. Example:

`76543210` position
`11010110` data bits

$x^7+x^6+x^4+x^2+x$ the polynomial corresponding to the given data bits

The principle of CRC: Sender and recipient agree upon a „divisor polynomial“, also called generator polynomial. Then, g zeros are added to the message, g being the degree of the generator polynomial. In the next step the sender divides the message (extended by g zeros), similar to polynomial division. In most cases there’ll be a remainder, the result of the division is of no interest. The remainder is then subtracted from the message extended by the zeros. The resulting bit string is now transferred to the recipient. If the message was transmitted correctly, then no remainder should emerge on the recipient’s side. Why? Because the sender intentionally subtracted the remainder before sending the message. The g zeros which emerge during the division are interpreted by the recipient as an indication of an error free transmission and aren’t processed any further (in particular the zeros are not data bits!)

Note: The sender can safely subtract the remainder without harming the message. Why?

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Error control: Cyclic Redundancy Check

The only difference to normal polynomial division: Calculations are binary and after the calculation of each digit a modulo 2 operation is performed! In other words: Always ignore the carry-over. This simplifies the addition and subtraction significantly.

```
 10101101    01001110
+01011100   -11101010
---------   ---------
 11110001    10100100
```

Discovery: Both operations, plus and minus, are equivalent to the XOR.

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Error control: Cyclic Redundancy Check (CRC)

Example:

`Message:` 1101011011
`Generator polynomial: 10011` (x^4+x+1) = 4th degree (5th order)
`Message extended by 4 zeros:` 11010110110000

```
Division:11010110110000:10011=  (the result doesn't matter)
     XOR 10011
        ------
         010011
     XOR -10011
         ------
              010110
          XOR -10011
              ------
              00010100
             XOR 10011
                 001110 = remainder
```

Special note: The division continues, if the MSB (most significant bit) of the divisor and of the bit string currently being divided is set. Sometimes the bit string has to be extended by new bits (from the message) until the generator polynomial "fits under it".

```
  11010110110000
minus (XOR) 1110
  11010110111110 =
```

transmitted message, which should generate no remainder when divided

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Error control: (CRC) – Recognized errors

Which errors are being recognized?

For the following analysis separate the error from the message:

Transmitted message including an error (or the corresponding polynomial) : M(x)
Original message (error free) : T(x)

Separate the transmitted message in: M(x) = T(x)+E(x)

with E(x) representing the isolated error. Every bit which is set in E stands for a toggled bit in M. A sequence from the first set bit to the last is called a burst-error. A burst-error can occur anywhere in E.

Question: Is the following calculation divided by the generator polynomail G(x) without a remainder? If so, then we can’t detect the error.

`[T(x)+E(x)]:G(x) = „remainder-less“?`

T(x):G(x) is divisible without any remainder, because we constructed the message exactly for this purpose. So, the analysis is reduced to the question whether E(x):G(x) erroneously results in no remainder.

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Error control: (CRC) – Recognised errors

Which errors are recognized?

1-bit errors:

The burst consists of only one error. If the generator polynomial has more than one coefficient, E(x) with a leading 1 followed by zeros cannot be divided without a remainder.

Example:

```
 1000(...)0:101=1
-101
 ---
 001 ... continued as above...
```

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Error control: (CRC) – Recognised errors

Which errors are recognized?

2-bit error:

A 2-bit error must look like this: $x^i+(...)+x^j$, so x^j can be factored out.
This results in $x^j(x^{(i-j)}+1)$.

It has already been proven that a generator polynomial with more than one term can't divide the factor x^j. When is a term (x^k+1) divided? (with k=i-j)

For a given generator polynomial this has to be tested for 2-bit bursts with different lengths. Here, the error has (inevitably) always the form 10(...)01.

What follows is an example program to test whether the generator polynomial

$x^{15}+x^{14}+1$

is useful regarding 2 bit errors.

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Error control: (CRC) – Recognized errors

Which errors are recognized?

```
main()
{
  char* generator = "1100000000000001";
  char* bit_string;

  for(int length = 2; length < 60000; length++) {

    if((length % 100) == 0) cout << length << endl;
    bit_string = new char[length+1];

    for(int j = 1; j < length-1; j++) // clear bitstring
      bit_string[j] = '0';

    bit_string[0] = '1'; bit_string[length-1] = '1'; bit_string[length] = 0;

    // test if divisible by generator polynomial
    if(Divisible(bit_string, length, generator, strlen(generator)) == true) {
      cout << "Division successful with length " << length << endl;
      break;
    } // if

    delete[] bit_string; bit_string = NULL;
  } // for

  if(bit_string) delete[] bit_string;
} // main
```

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Error control: (CRC) – Recognized errors

Which errors are recognized?

Error polynomials with an odd number of terms:

Speculation: If the generator polynomial contains the term (x^1+x^0), an error string with an odd number of bits can't be divided.

Proof by contradiction: Assuming E(x) being divisible by (x^1+x^0), the factor can also be extracted:

$$E(x) = (x^1+x^0)Q(x)$$

So far we only divided polynomials. Now we use it as a function and evaluate it for x=1.

(x^1+x^0) equals $(1+1)$ and $Q(x)$ equals 1, because $Q(x)$ still contains an odd number of terms (additions are still done modulo 2). Hence follows $(1+1)Q(x)=0x1=0$

But in the beginning we assumed that $E(x)$ contains an odd number of terms. So the result should have been 1, not 0. As follows, the factor (x^1+x^0) can't be extracted. As a consequence $E(x)$ isn't divisible by (x^1+x^0), if it contains an odd number of terms (or error bits).
Result: The generator polynomial should contain the term (x^1+x^0) to catch all errors with an odd number of bits.

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Error control: (CRC) – Recognized errors

Which errors are recognized?

Recognition of burst errors of length r:

The burst error in E(x) could look like this: `0001 anything 100000`

To move the last bit to the right, a factor can be factored out:

$E(x)=x^i(x^{(r-1)}+ ... + 1)$

with i being the number of zeros right of the last 1.

If the degree of the generator polynomial itself is r (so it has r+1 terms), the factor $(x^{(r-1)}+ ... + 1)$ also cannot be divided, because the generator polynomial is larger than the number to be divided.

Example (decimal system): 99:1234567 isn't divisible without a remainder (result 0, remainder 99). Detecting burst errors with length r is trivial in the way that the error itself simply occurs at the end of the division.

Even if the burst is just as large as the generator polynomial, the division yields a remainder only if by chance the error coincides exactly with the generator polynomial. This is possible, but not very likely.

Example (decimal system): 1234567:1234567=1 (modulo 0)

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Pure ALOHA

Idea: Everybody may transmit whenever desired, but only a frame of max. length. Other participants who are already sending aren't regarded.

Collision:

If two frames overlap, both of them are considered as destroyed. But both participants can detect that there was a collision and send their message (frame) once again. Of course both frames would collide again, so there would never be a valid transmission.

Solution:

Every sender waits for a random amount of time before starting its transmission (in the case of sensor nodes this time could be „overslept" to save on energy). The solution to the collision problem is that the participant with the shorter delay has (hopefuly) already finished its transmission before the one with the longer delay starts to send. In the worst case a repeated collision will happen and the process has to be repeated.

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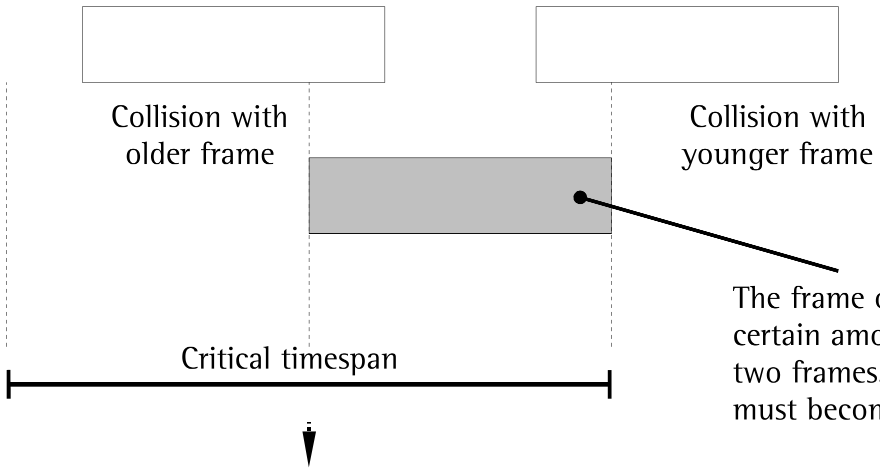
Classic medium access control (MAC) protocols for sensor nodes?

Pure ALOHA

The frame can collide on both ends:



Collision with older frame

Collision with younger frame

Critical timespan

The frame of a sender has to be „left alone“ for a certain amount of time, which is at least as long as two frames. During this timespan no other sender must become active.

Event of „special recklessness*“. If the second (gray) sender listened, it could avoid the collision and would not harm its own packet and the first sender’s packet.

*rücksichtslos

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Pure ALOHA: Analytical review

Arrival rate λ	Average number of attempts to access the channel. If there are n stations each of which produces an individual arrival rate g, the overall rate λ=ng.
Transmission rate S	Number of frames that actually reach their destination. S = 0: no frame reaches its destination because... a) no frames are sent b) so many frames are sent that all of them collide S = 1: every frame can be transmitted without collision, e. g. because there's only one sender
P_S	Probability that frames are transmitted successfully
$S = \lambda P_S$	Apparently there are two extremes: a) The senders create hardly any arrivals (λ=0). This result in good conditions for the transmission, because the medium is always free (P=1), but it is also not used. b) The senders create many arrivals (λ large). This results in continuous collisions (P=0), so nothing is transmitted without collision. The optimum lies in between.

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Pure ALOHA: The Poisson distribution

λ is the arrival rate per time unit. The probability for n arrivals per time unit is calculated like this:

$$P_\lambda(n)=\frac{\lambda^n}{n!}e^{-\lambda}$$

λ = number of arrivals per time interval
$1/\lambda$ = mean time between two consecutive arrivals

n = number of arrivals, for which we want to obtain the probability P(n)

Which n will yield the highest probability?

Remark: The use of the Poission distribution requires an exponential distribution of the time of arrival in between two consecutive events. In most cases of mutually independent results and a large number of participants this can be assumed.

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Pure ALOHA: Analytical review

Example: We know that a department in a store registers an average number of two arrivals of customers per minute during lunchtime. What are the probabilities for 0, 1, 2 etc. customers, resp. that at least 0, 1, 2, etc. customers arrive?

$\lambda = 2$, n=0,1,2, ...

n	P(n)	Sum	
0	0,135	0	0,135
1	0,271	0	0,406
2	0,271	0	0,677
3	0,180	0	0,857
4	0,090	0	0,947
5	0,036	0	0,983
6	0,012	0	0,995
7	0,003	0	0,998

One salesperson has nothing to do with a P' of 13%.

With P'=40%, one salesperson is enough for customer care.

With W'=59% there's a need for at most 2 salesperson.

In 32% of the cases at least a third salesperson would be necessary.

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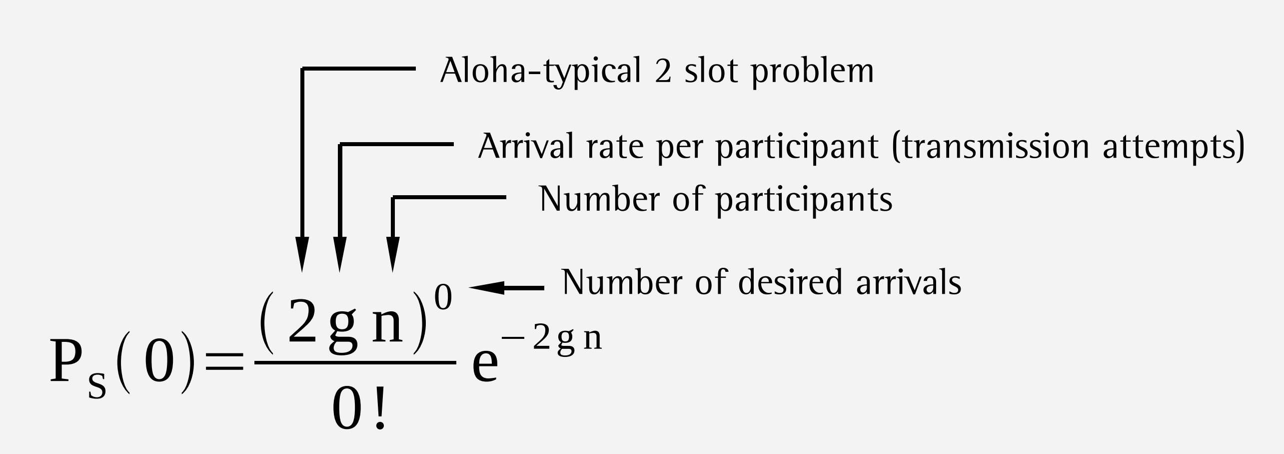
Pure Aloha: Analytical review

As seen before a successfully transmitted packet needs at least two frames. In other words there have to be exactly 0 arrivals of other packets during two periods.



Aloha-typical 2 slot problem

Arrival rate per participant (transmission attempts)

Number of participants

Number of desired arrivals

$$P_S(0)=\frac{(2gn)^0}{0!}e^{-2gn}$$

How does the probability P_S of the chance to send successfully vary with the number of participants?

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Pure ALOHA: Analytical review

Packet throughput of pure Aloha

On the last page we calculated the probability of the channel being idle, when there's an certain average number of packets to be sent (the arrival rate).

In fact, the achievable data throughput L (having n participants with a transmission rate of g packets per frame length and participant) is more interesting.

Arrival rate (in packets per frame length) on the channel

$$L(n,g)=gn\frac{(2gn)^0}{0!}e^{-2gn}$$

Probability of success

What's the maximum packet throughput achievable with classic ALOHA?

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Pure ALOHA: Analytical review

Probability of successful transmission on the kth attempt to send:

$$P(k=K)=(1-P_S)^{k-1}P_S$$

One try succeeded

k-1 attempts failed

Average time to deliver one packet:

$$D=\sum_{k=1}^{\infty} k(1-P_S)^{k-1}P_S$$

Despite the (k-1) waiting periods the transmission isn't safe at attempt number k (Why? Because for the other cases of k there must be some „probability left").

If the packet is sent after exactly k frames, it has been waiting (k-1) times before.

Of course, if k frames are necessary to transmit the packet, they have to pass, but only with the following probability

For the sender a different number k of frames could occur. The packet can't be transmitted faster than after a full frame length. In this case no waiting period would have occurred.

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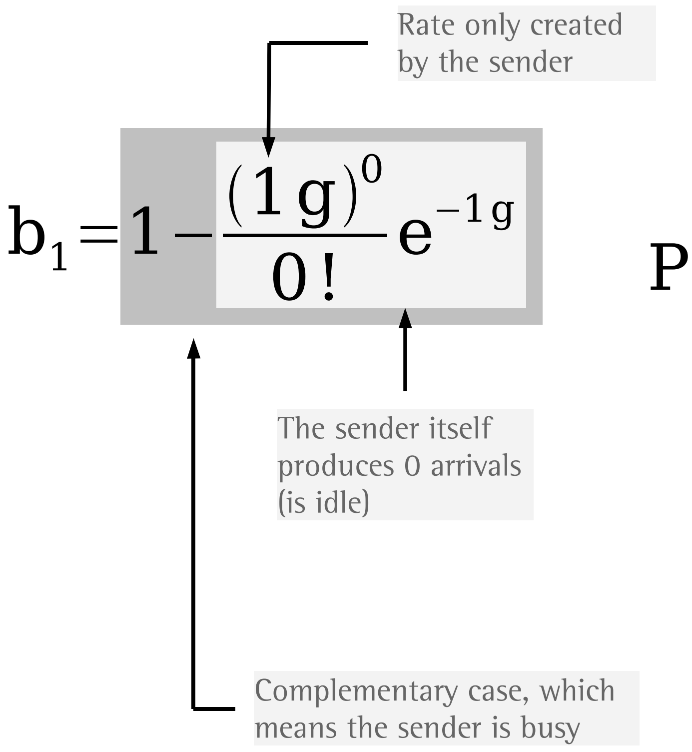
Classic medium access control (MAC) protocols for sensor nodes?

Energy efficiency of pure Aloha

Because the medium is always monitored in Aloha whenever there's nothing to send, the energy consumption alternates only between P_{TX} and P_{RX}.

P_{RX}	Energy consumption during reception
P_{TX}	Energy consumption during transmission

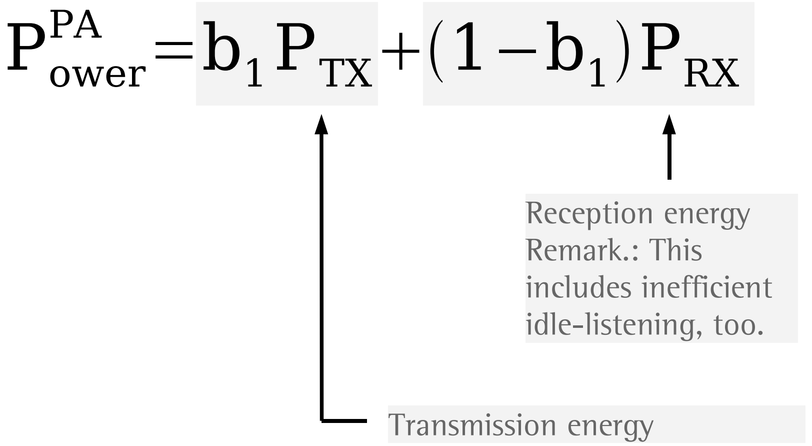


$$b_1 = \boxed{1 - \boxed{\frac{(1g)^0}{0!} e^{-1g}}}$$

Rate only created by the sender

The sender itself produces 0 arrivals (is idle)

Complementary case, which means the sender is busy



$$P^{PA}_{ower} = b_1 P_{TX} + (1 - b_1) P_{RX}$$

Reception energy
Remark.: This includes inefficient idle-listening, too.

Transmission energy

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Classic medium access control (MAC) protocols for sensor nodes?

Energy efficiency of classic Aloha

Example

$P_{\text{base consumption}}$ = 8 mA
P_{TX} = 20 mA
P_{RX} = 6 mA

$$b_1 = 1 - \frac{(1\,g)^0}{0\,!} e^{-1\,g}$$

$$P^{PA} = b_1 P_{TX} + (1 - b_1) P_{RX}$$

A frame length is 10ms, i. e. 100 frames/second.
Nodes will send once per second.

g=0,01 => b_1 = ~(1-0,99) = 0,01

P^{RA} = 0,01x20 + (1-0,01)x6 + 8 = ~14,14 mA

Battery with 2000 mAh will last for 2000x60x60/14,14 = ~509193s = ~141 hours

Result: For hardly 6 days no sensor nodes can be scattered into a field.

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Example for medium access control (MAC) protocols:

Pure aloha for sensor networks?

\+ Sender can send at anytime. In particular it doesn't have to be „awake" before (at least not prior to sending). Instead, it sends whenever there's a need.

\+ If the transmission failed, the randomly determined waiting period has to pass. During this waiting period the node can switch to energy saving mode.

\- The node would basically always have to listen, when a message is expected. Especially in the field of long-distance routing it is uncertain in advance when a message has to be received and forwarded. That is because in this case the node isn't the initiator but the „service provider".

Permanent readiness to receive is not an option for sensor nodes. The problem has to be solved more energy efficiently.

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Simulation of MAC-protocols

```
const long MAX_TIME             = 10000L;
const long NO_STATIONS          = 100L;
const long FRAME_LENGTH         = 100L;
      long medium_occupied_till = -1;
      long survival_timer       = -1;
      long successful_packets   = 0;
      long no_frames            = MAX_TIME / FRAME_LENGTH;

class Station
{
public:
  void Init() {};
  void TriggerSend(long);
}; // class Station

void Station::TriggerSend(long current_time)
{
  if(current_time < medium_occupied_till) {
    survival_timer = -1;
  } // if
  else survival_timer = 0;

  medium_occupied_till = current_time + FRAME_LENGTH;
} // Station::TriggerSend
```

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Classic medium access control (MAC) protocols for sensor nodes?

Simulation of different MAC-protocols

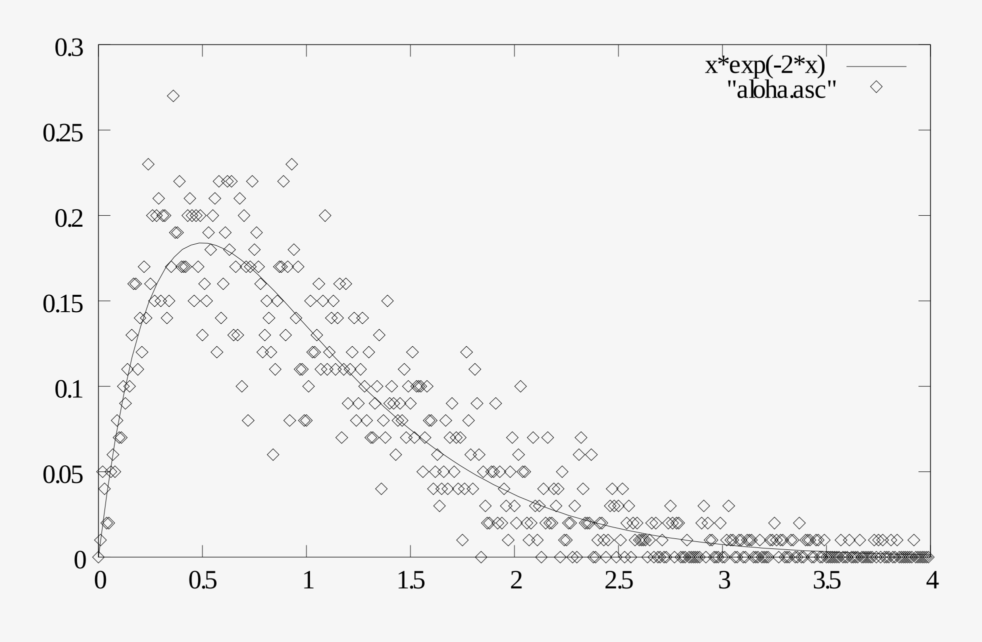
```
main()
{
  Station* station          = new Station[NO_STATIONS];

  for(long i = 0; i < NO_STATIONS; i++)
    station[i].Init();
  for(long arrival_rate = 0; arrival_rate < 400; arrival_rate += 1) {
    medium_occupied_till = -1; survival_timer = -1; successful_packets = 0;
    for(long time = 0; time < MAX_TIME; time++) {
      for(long station_index = 0; station_index < NO_STATIONS; station_index++)
        if((abs(rand()) % (100*NO_STATIONS*FRAME_LENGTH)) < arrival_rate)
          station[station_index].TriggerSend(time);

      if(survival_timer != -1) survival_timer++;
      if(survival_timer == FRAME_LENGTH) {
        successful_packets++;
        survival_timer = -1;
      } // if
    } // for

    double overall_arrival_rate = ((double)arrival_rate)/100.0;
    cout << overall_arrival_rate << " "
         << successful_packets/(double)no_frames << endl;
  } // for

  delete[] station;
} // main
```

Communication in sensor networks

Classic medium access control (MAC) protocols for sensor nodes?

Simulation of different MAC-protocols

After the analytic view the Aloha approach is now to be simulated using the program on the previous page.



With 100 stations, 100 frames, each divided into 100 simulation time units the distribution fairly adapts to the curve. Why do in practice higher or lower data throughputs still occur in some simulation runs?

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