2.2 Still Image Compression Techniques

2.2.1 Telefax

New *telecommunication* standards are defined by the International Telecommunications Union (ITU-T) (in former times: CCITT = Commitée Consultatif International de Téléphonie et Télégraphie).

The standard for lossless Telefax compression was one of the early standards for still image compressions.

Images are interpreted by the Group 3 compression standard as two-tone (black-and-white) pictures. As a result a pixel can be represented by one bit. The example shows a part of a line of black-and-white pixels. Obviously, runs will be much larger than 1 in most cases, and thus run-length encoding is efficient.

Example:

run-length encoding: 4w 3b 1w 1b 2w 1b

Fax Standards of ITU-T

Standard T.4

First passed in 1980, revised in 1984 and 1988 (Fax Group 3) for error-prone lines, especially telephone lines

- Two-tone (black-and-white) images of size A4 (similar to US letter size)
- Resolution: 100 dots per inch (dpi) or 3,85 lines/mm vertical, 1728 samples per line

Objective:

Transmission at 4800 bits/s over the telephone line (one A4 page per minute)

Standard T.6

First passed in 1984 (Fax Group 4) for error-free lines or digital storage.

Compression Standards for Telefax (1)

Telefax Group 3, ITU-T Recommendation T.4

First approach: Modified Huffman Code (MH)

- Every image is interpreted as consisting of lines of pixels
- For every line the run-length encoding is calculated.
- The values of the run-length encoding are Huffman-coded with a standard table.
- Black an white runs are encoded using different Huffman codes because the run length distributions are quite different.
- For error detection an EOL (end-of-line) code is inserted in the end of every line. This enables re-synchronization in case of bit transmission errors.

Compression Standards for Telefax (2)

Second approach: Modified Read (MR) Code

- The pixel values of the previous line are used to predict the values of the current line
- Then run-length encoding and a static Huffman code are used (same as for MH).
- The EOL code is also used.

The MH and MR coding alternates in order to avoid error propagation.

Huffman-Table for Telefax Group 3 (excerpt)

White run length	Code word	Black run length	Code word
0	0 0 1 1 0 1 0 1	0	0 0 0 0 1 1 0 1 1 1
1	0 0 0 1 1 1	1	0 1 0
2	0 1 1 1	2	1 1
3	1 0 0 0	3	1 0
4	1 0 1 1	4	0 1 1
5	1 1 0 0	5	0 0 1 1
6	1 1 1 0	6	0 0 1 0
7	1 1 1 1	7	0 0 0 1 1
8	1 0 0 1 1	8	0 0 0 1 0 1
9	1 0 1 0 0	9	0 0 0 1 0 0
1 0	0 0 1 1 1	1 0	0 0 0 0 1 0 0
1 1	0 1 0 0 0	1 1	0 0 0 0 1 0 1
1 2	0 0 1 0 0 0	1 2	0 0 0 0 1 1 1
1 3	0 0 0 0 1 1	1 3	0 0 0 0 0 1 0 0
1 4	1 1 0 1 0 0	1 4	0 0 0 0 0 1 1 1
1 5	1 1 0 1 0 1	1 5	0 0 0 0 1 1 0 0 0
1 6	1 0 1 0 1 0	1 6	0 0 0 0 0 1 0 1 1 1
1 7	1 0 1 0 1 1	1 7	0 0 0 0 0 1 1 0 0 0
1 8	0 1 0 0 1 1 1	1 8	0 0 0 0 0 0 1 0 0 0
1 9	0 0 0 1 1 0 0	1 9	0 0 0 0 1 1 0 0 1 1 1
2 0	0 0 0 1 0 0 0	2 0	0 0 0 0 1 1 0 1 0 0 0

Telefax Group 4

Telefax Group 4, ITU-T Recommendation T.6

Coding techniques: Modified Modified Read Code (MMR)

- Simplification of the MR code; there are no error detection mechanisms in order to improve the compression rate.

Typical compression rates:

	business documents
Group 3:	20:1
Group 4:	50:1

For photos (and the like) the compression rate of T.6 is low because the the lengths of the runs are very short. Other schemes such as arithmetic coding would be more suitable.

2.2.2 Block Truncation Coding (BTC)

This simple coding algorithm is used in the compression of monochrome images. Every pixel is represented by a grey value between 0 (black) and 255 (white).

The BTC Algorithm

1. Decompose the image into blocks of size n x m pixels.
2. For each block calculate the mean value and the standard deviation as follows:

$$\mu = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m} Y_{i,j} \qquad \sigma = \sqrt{\frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}(Y_{i,j}-\mu)^2}$$

where $Y_{i,j}$ is the brightness of the pixel.

3. Calculate a bit array B of size n x m as follows:

$$B_{i,j} = \begin{cases} 1 \ldots & if \quad Y_{i,j} \le \mu \\ 0 \ldots & else \end{cases}$$

The BTC Algorithm (continued))

4. Calculate two grey scale values for the darker and the brighter pixels:

$$a = \mu - \sigma\sqrt{p/q}$$

$$b = \mu + \sigma\sqrt{q/p}$$

p is the number of pixels having a larger brightness than the mean value of the block, q is the number of pixels having a smaller brightness.

5. Output: (bit matrix, *a, b*) for every block.

Decompression with BTC

For every block the grey value of each pixel will be calculated as follows:

$$Y'_{i,j} = \begin{cases} a \ldots & if \\ b \ldots & else \end{cases} \qquad B_{i,j} = 1$$

Compression rate example

Block size :	4 x 4
Original (grey values)	1 byte per pixel
Encoded representation:	bit matrix with 16 bits + 2 x 8 bits for *a* and *b*

=> reduction from 16 bytes to 4 bytes, i.e., the compression rate is 4:1.

2.2.3 Color Cell Compression

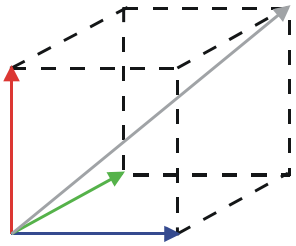
Color cell compression (CCC) is a algorithm for the compression of color images. In principle, BTC can be used for color images rather than for grey scale images by compressing the three color components separately. However, the Color Cell Compression algorithm leads to a better compression rate.

Color Models

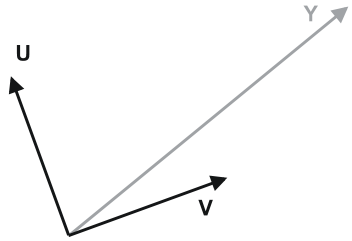
The classical color model for the computer is the **RGB model**. The color value of a pixel is the sum of the intensities of the color components red, green and blue. The maximum intensity of all three components results in white.

In the **YUV model, Y** represents the value of the luminance (brightness) of the pixel, U and V are two orthogonal color vectors.

The color value of an pixel can be easily converted from model to model.



RGB-Model



YVU-Model

An advantage of the YUV model is that the value of the luminance is directly available. That means that a grey scale version of the image can be created very easily. Another point is that the compression of the luminance component can differ from the compression of the chrominance components.

The CCC Algorithm

1. Decompose the image into blocks of size n x m pixels.
2. The brightness of a pixel is computed as follows:

$$Y = 0.3P_{red} + 0.59P_{green} + 0.11P_{blue}$$

Y=0 is equivalent to black, Y=1 is equivalent to white

3. For c = red, green, blue calculate the mean color value of the pixel as follows:

$$a_c = \frac{1}{q} \sum_{Y_{i,j} \le \mu} P_{c,i,j}\ , \qquad b_c = \frac{1}{p} \sum_{Y_{i,j} \succ \mu} P_{c,i,j}$$

Again, *q* and *p* are the numbers of pixels with a brightness larger or smaller than the mean value, respectively.

The CCC Algorithm (continued)

4. Calculate a bit array B of size n x m as follows:

$$B_{i,j} = \begin{cases} 1 \ldots & if \quad Y_{i,j} \le \mu \\ 0 \ldots & else \end{cases}$$

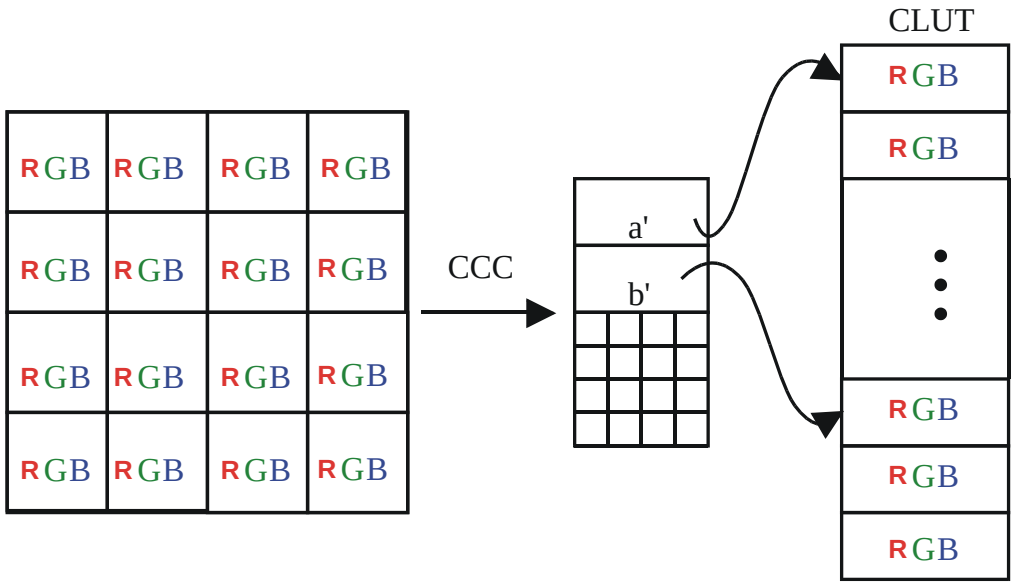
5. The color values *a* = (a_{red}, a_{green}, a_{blue}) and *b* = (b_{red}, b_{green}, b_{blue}) are now quantized onto a color lookup table. We get the values *a'* and *b'* as an index for the Color Lookup Table (CLUT).
6. Output: (bit matrix, *a'*, *b'*) for every block

Decompression of CCC Images

For every block the decompression algorithm works as follows:

$$P'_{i,j} = \begin{cases} CLUT\ [a'] \ldots & if \quad B_{i,j} = 1 \\ CLUT\ [b'] \ldots & else \end{cases}$$

Usage of the Color Lookup Table in CCC

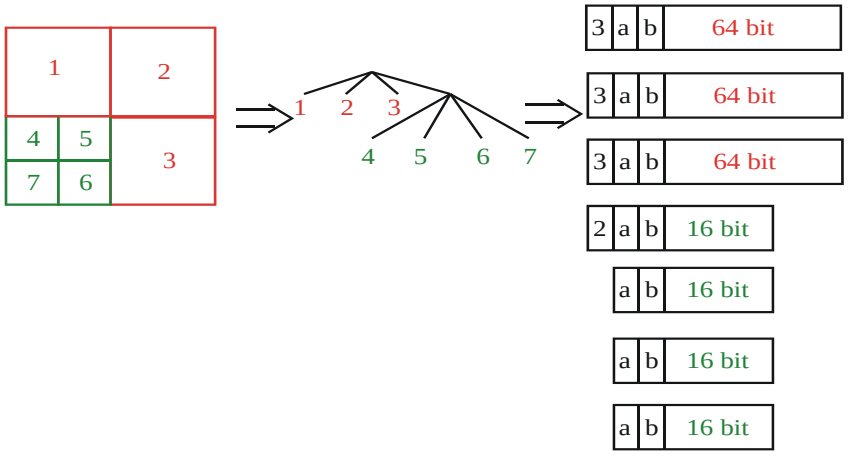


Extended Color Cell Compression (XCCC)

This method is an extension of CCC to further improve the compression rate.

Idea

Use a hierarchy of block sizes. In the first step the algorithm tries to code a large block with CCC. If the difference to the true color values is greater than a given threshold the block is divided into four parts. The algorithm works recursively (invented at U. Mannheim).



2.2.4 A Brief Introduction to Transformations

Motivation for Transformations

Improvement of the compression ratio while maintaining a good image quality.

What is a transformation?

- Mathematically: a change of the base of the representation
- Informally: representation of the same data in a different way.

Motivation for the use of transformations in compression algorithms: **In the frequency domain, leaving out detail is often less disturbing to the human visual (or auditive) system than leaving out detail in the original domain.**

The Frequency Domain

In the frequency domain the signal (one-dimensional or two-dimensional) is represented as an overlay of base frequencies. The coefficients of the frequencies specify the amplitudes with which the frequencies occur in the signal.

The Fourier Transform

The Fourier transform of a function *f* is defined as follows:

$$\hat{f}(t) = \int f(x) e^{-2\pi i t x} dx$$

where *e* can be written as

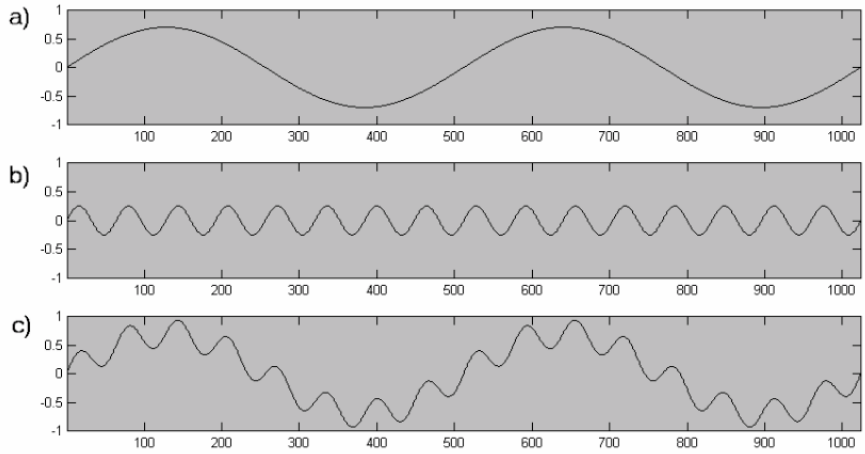
$$e^{ix} = \cos(x) + i \sin(x)$$

Note:

The *sin* part makes the function complex. If we only use the *cos* part the transform remains real-valued.

Overlaying the Frequencies

A transform asks how the amplitude for each base frequency must be chosen such that the overlay (sum) best approximates the original function.



The output signal (c) is represented as a sum of the two sine waves (a) and (b).

One-Dimensional Cosine Transform

The Discrete Cosine Transform (DCT) is defined as follows:

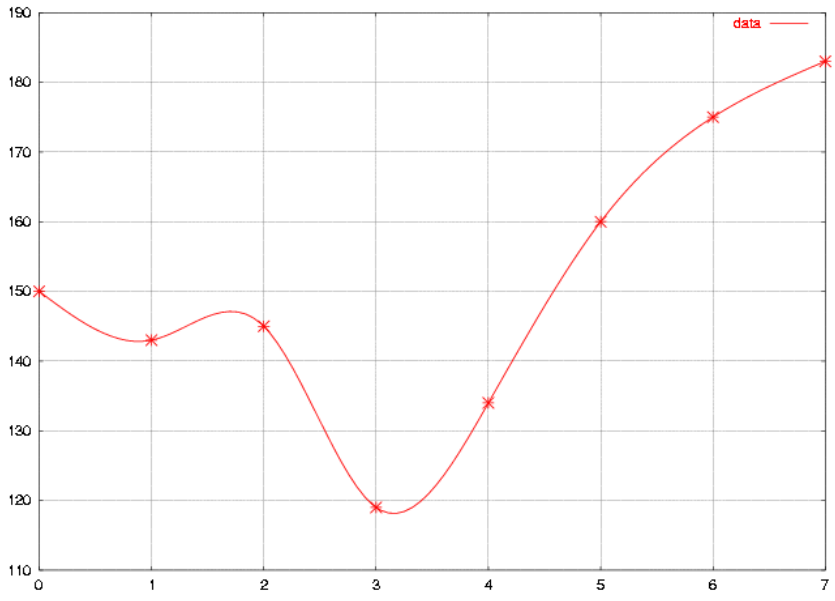
$$S_u = \frac{1}{2} C_u \sum_{x=0}^{7} s_x \cos \frac{(2x+1)u\pi}{16}$$

with

$$C_u = \begin{cases} \frac{1}{\sqrt{2}} & for\ u = 0 \\ 1 & otherwise \end{cases}$$

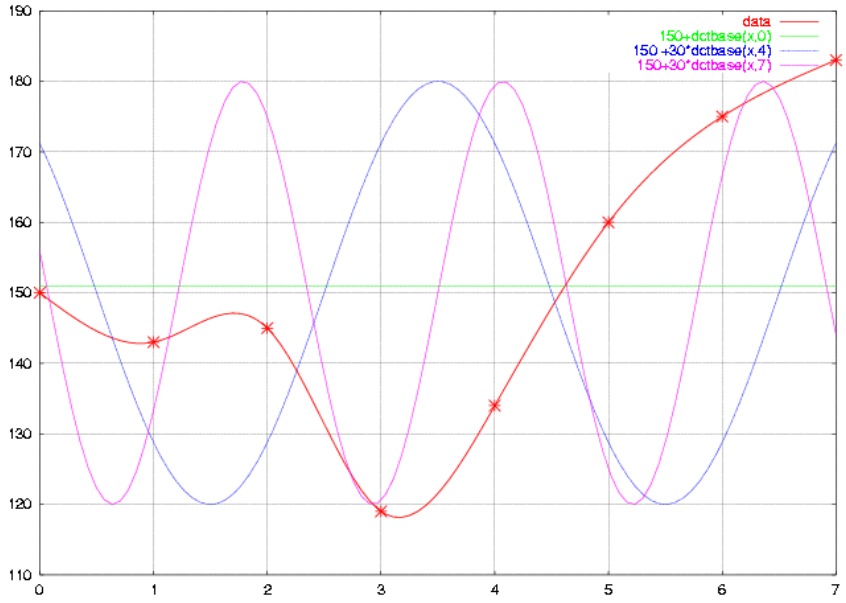
Example for a 1D Approximation (1)

The following one-dimensional signal is to be approximated by the coefficients of a 1D–DCT with eight base frequencies.



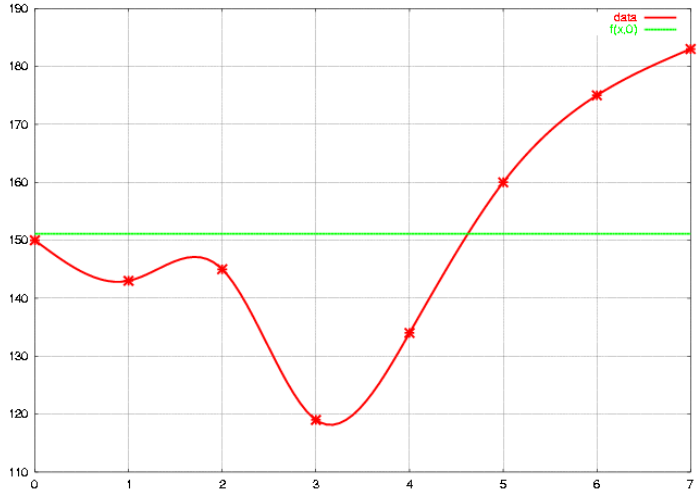
Example for a 1D Approximation (2)

Some of the DCT kernels used in the approximation.



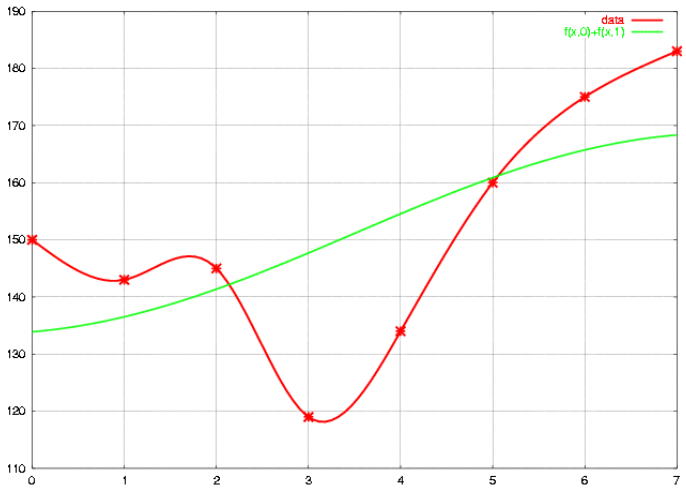
Example for a 1D Approximation (3)

DC coefficient



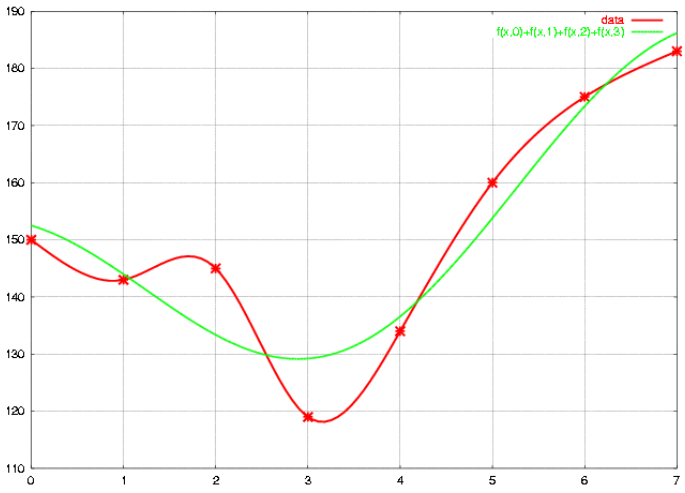
Example for a 1D Approximation (4)

DC coefficient + 1st AC coefficient



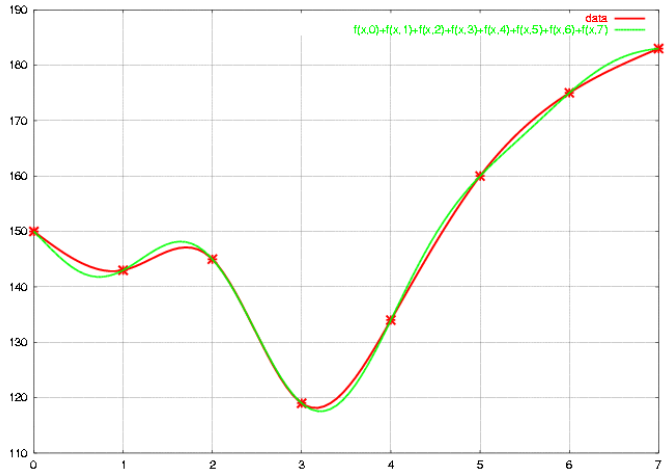
Example for a 1D Approximation (5)

DC coefficient + AC coefficients 1-3



Example for a 1D Approximation (6)

DC coefficient + AC coefficients 1-7



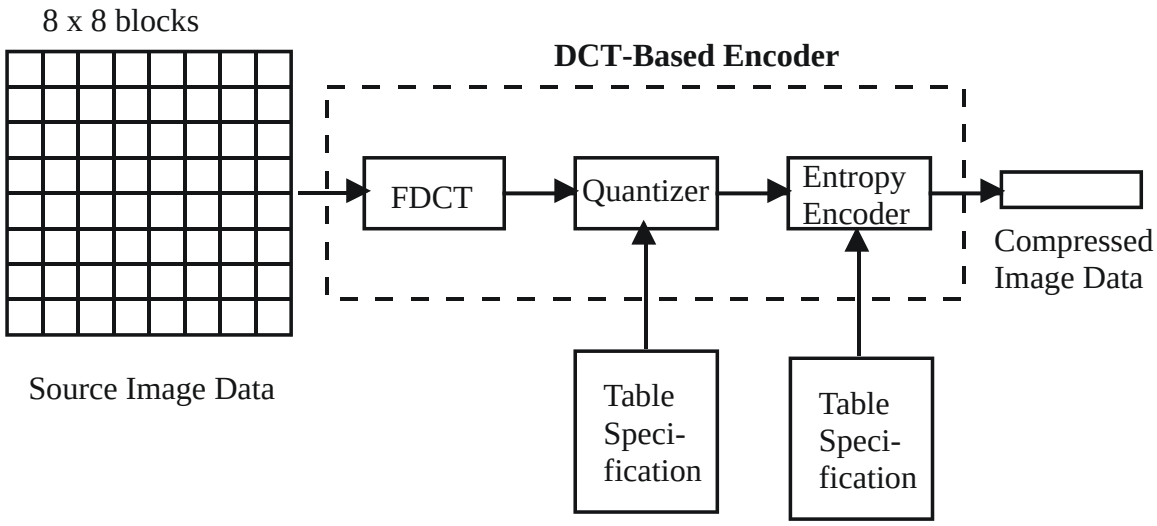
2.2.5 JPEG

The Joint Photographic Experts Group (JPEG, a working group of ISO) has developed a very efficient compression algorithm for still images which is commonly referred to under the name of the group.

JPEG compression is done in in four steps:

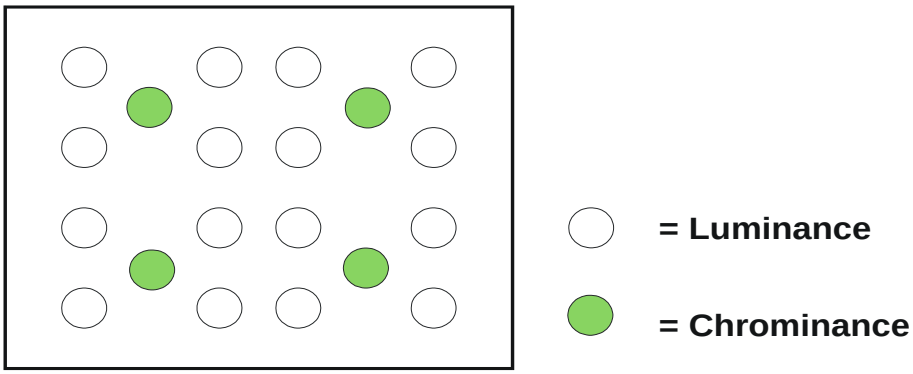
1. Image preparation
2. Discrete Cosine Transform (DCT)
3. Quantization
4. Entropy Encoding

The DCT-based JPEG Encoder



Coding of the Color Components with a Lower Resolution ("Color Subsampling")

One advantage of the YUV color model is that the color components U and V of a pixel can be represented with a lower resolution than the luminance value Y. The human eye is more sensitive to brightness than to variations in chrominance. Therefore JPEG uses **color subsampling:** for each group of four luminance values one chrominance value for each U and V is sampled.



In JPEG, four Y blocks of size of 8x8 together with one U block and one V block of size 8x8 each are called a macroblock.

JPEG "Baseline" Mode

JPEG Baseline Mode is a compression algorithm based on a DCT transform from the time domain into the frequency domain.

Image transformation

FDCT (Forward Discrete Cosine Transform). Very similar to the Fourier transform. It is used separately for every 8x8 pixel block of the image.

$$S_{vu} = \frac{1}{4} C_u C_v \sum_{x=0}^{7} \sum_{y=0}^{7} s_{yx} \cos\frac{(2x+1)u\pi}{16} \cos\frac{(2y+1)v\pi}{16}$$

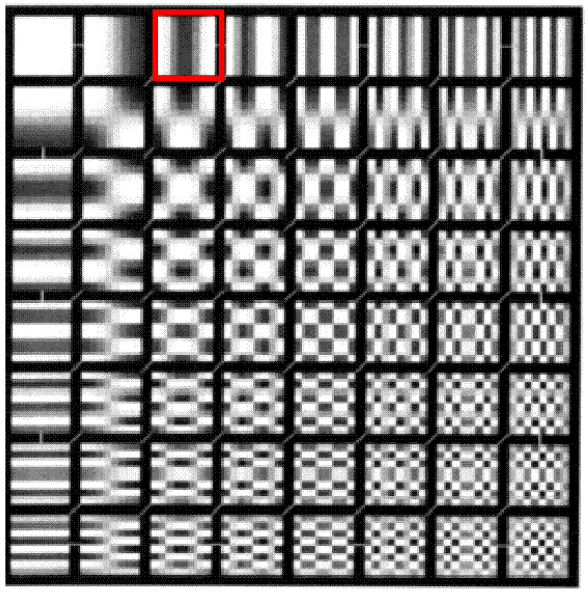
with

$$C_u, C_v = \begin{cases} \frac{1}{\sqrt{2}} & for\ u,v=0 \\ 1 & otherwise \end{cases}$$

This transform is computed 64 times per block. The result are 64 coefficients in the frequency domain.

Base “Frequencies” for the 2D-DCT

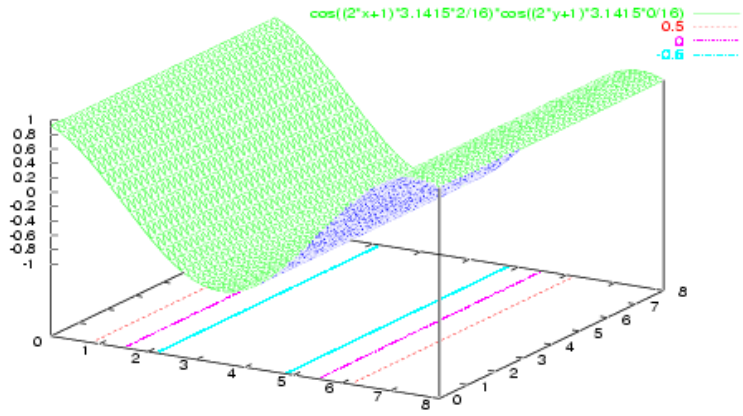
To cover an entire block of size of 8x8 we use 64 base “frequencies”, as shown below.



Example of a Base Frequency

The figure below shows the DCT kernel corresponding to the base frequency (0,2) shown in the highlighted frame (first row, third column) on the previous page.

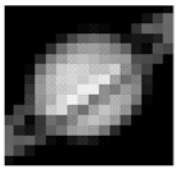
$$\frac{\cos(2x+1)\cdot 2\pi}{16}\cdot\frac{\cos(2y+1)\cdot 0\pi}{16}$$



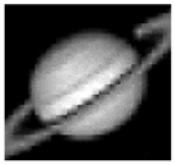
Example: Encoding of an Image with the 2D-DCT and block size 8x8

original

1 coefficient



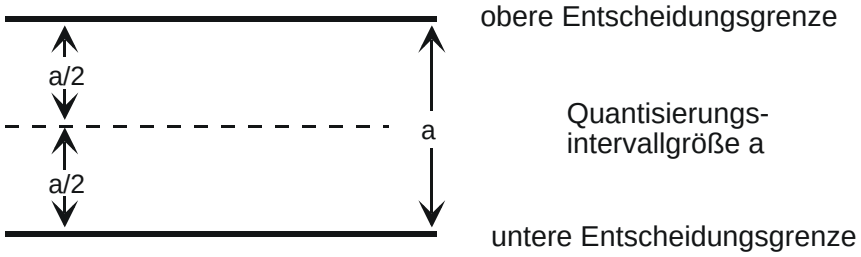
4 coefficients



16 coefficients

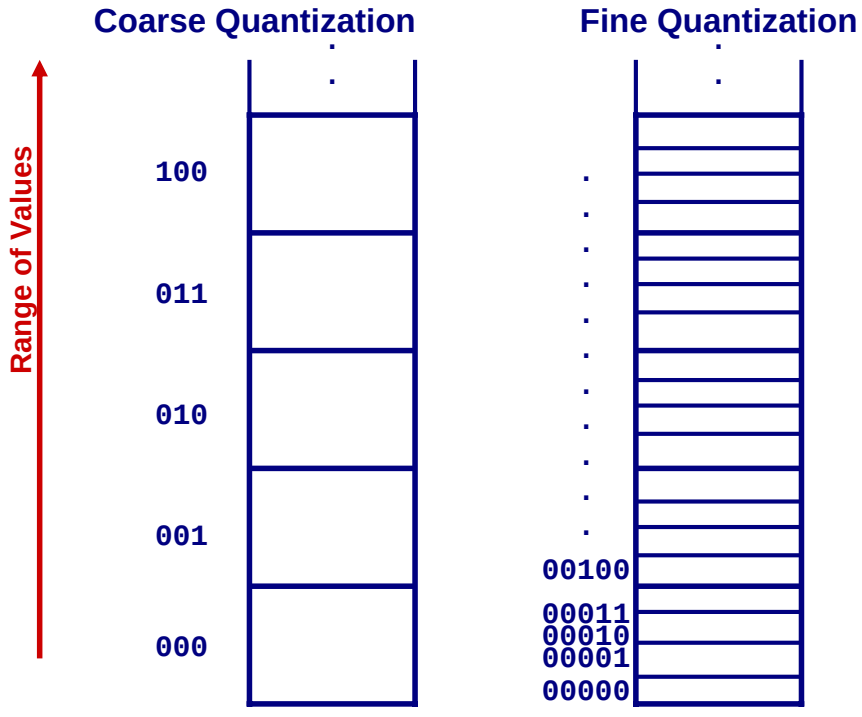
Quantization

The next step in JPEG is the quantization of the DCT coefficients. Quantization means that the range of allowable values is subdivided into intervals of fixed size. The larger the intervals are chosen, the larger the quantization error will be when we decompress.



Maximum quantization error: a/2

Quantization: Quality vs. Compression Ratio



Quantization

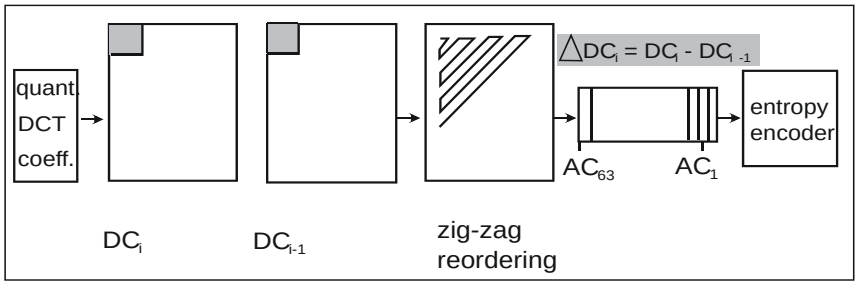
In JPEG the number of quantization intervals can be chosen separately for each DCT coefficient (Q-factor). The Q-factors are specified in a **quantization table**.

Entropy-Encoding

The quantization step is followed by an entropy encoding (lossless encoding) of the quantized values:

- The DC coefficient is the most important one (basic color of the block). The DC coefficient is encoded as the difference between the current DC coefficient value and the one from the previous block (differential coding).
- The AC coefficients are processed in zig-zag order. This places coefficients with similar values in sequence.

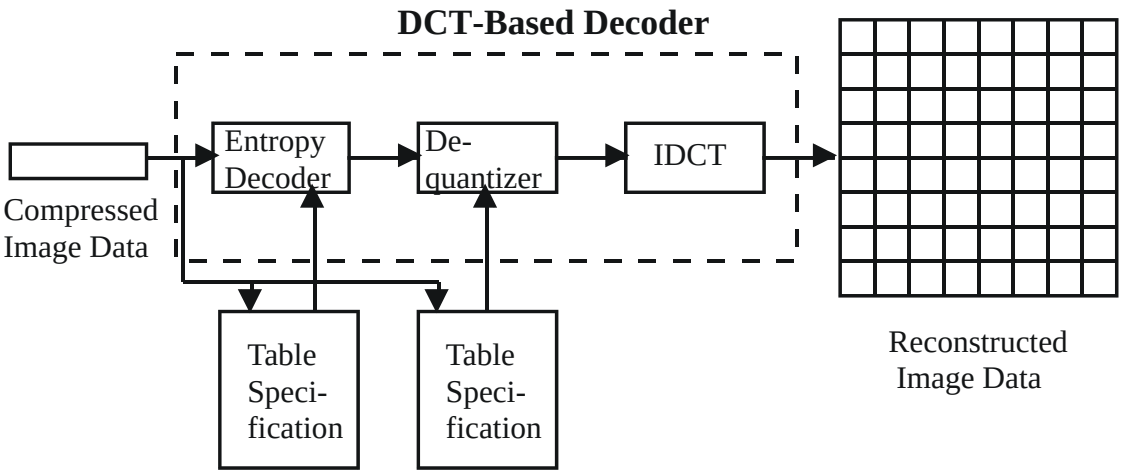
Quantization and Entropy Encoding



Zig-zag reordering of the coefficients is better than a read out line-by-line because the input to the entropy encoder will have a few non-zero and many zero coefficients (representing higher frequencies, i.e., sharp edges). The non-zero coefficients tend to occur in the upper left-hand corner of the block, the zero coefficients in the lower right-hand corner.

Run-length encoding is used to encode the values of the AC coefficients. The zig-zag read out maximizes the run-lengths. The run-length values are then Huffman-encoded (this is similar to the fax compression algorithm).

JPEG Decoder



Quantization Factor and Image Quality

Example: Palace in Mannheim

Palace, original image

Palace image with Q=6

Palace Example (continued)

Palace image with Q=12

Palace image with Q=20

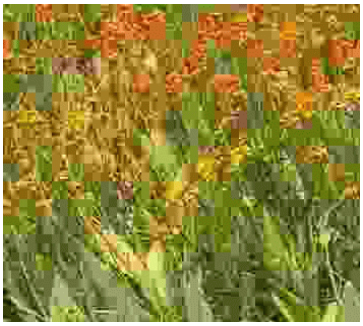
Flower Example (1)

Flower, original image

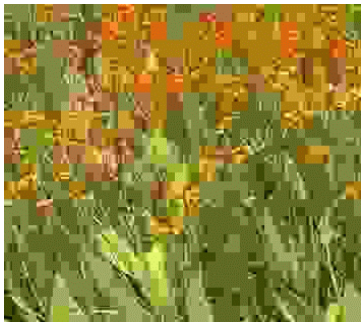
Flower with Q=6

Flower Example (2)

Flower with Q=12



Flower with Q=20



2.2.6 Compression with Wavelets

Motivation

Signal analysis and signal compression.

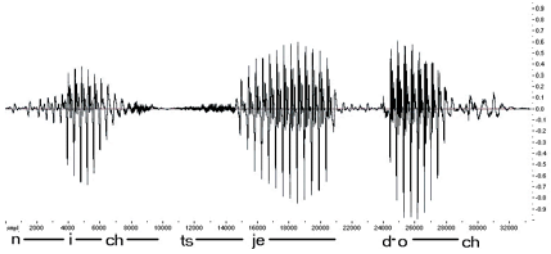
Known: image compression algorithm

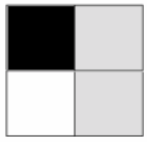
- based on the pixel values (BTC; CCC; XCCC)
- based on transformation in the frequency domain (Fourier Transform, DCT)

Examples

"Standard" representation of a signal:

- Audio signal with frequencies over the time
- Image as pixel values on locations





=

0	224
255	144

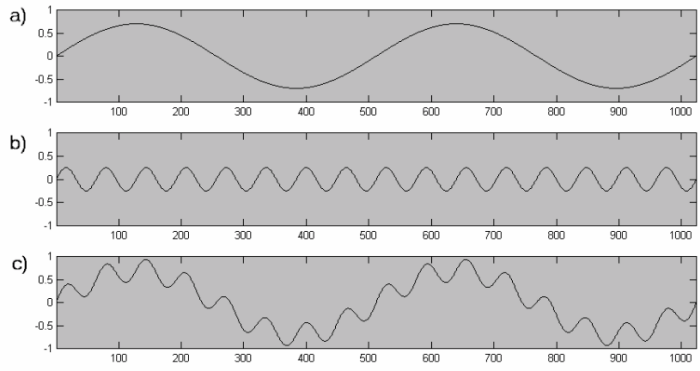
The Frequency Domain

In the Frequency domain the *changes* of a signal are our focus.

- How strong is the variation of the amplitude of the audio signal?
- How strong varies one pixel from the next?
- Which frequencies are contained in the given signal?

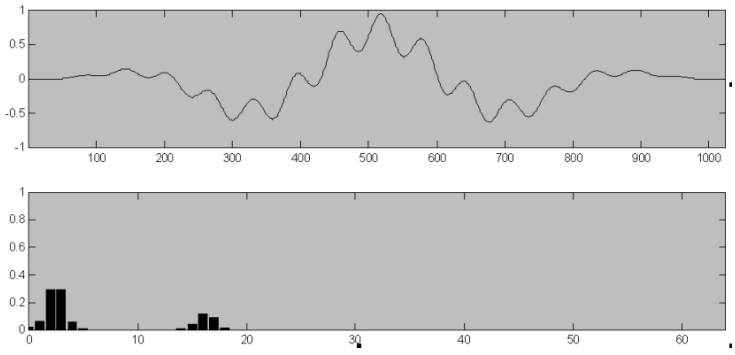
Time to Frequency Transformations

A transformation weights every single frequency to prepare it for a accumulation of all frequencies for to reconstruction of the original signal.



The output signal (c) is represented as a sum of the two sine waves (a) und (b).

Problems with the Fourier-Transformation



If we look at a signal with a high “locality”, a great many sine and cosine oscillations must be added. The example shows a signal (upper figure), which disappears on the edges. It is put together with sine oscillations from 0-5 Hz and 15-19 Hz (lower figure).

Wanted: A frequency representation based on functions that feature a high locality, i.e., are NOT periodic.

The solution: **Wavelets**

What is a Wavelet ?

A wavelet is a function ψ that satisfy the following permissibility condition:

$$0 < c_\psi := 2\pi \int_R \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

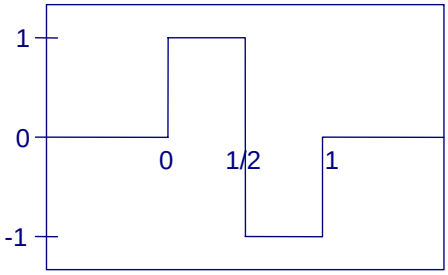
As follows:

$$0 = \hat{\psi}(0) = \int \psi(x) e^{-2\pi i 0 x} dx = \int \psi(x) dx$$

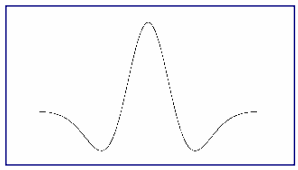
A wavelet is a function that exists only in a limited interval.

Example Wavelets

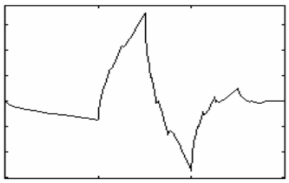
Haar Wavelet



Mexican Hat



Daubechies-2



Practical Application

We limit the further discussion to
- Discrete Wavelet Transformations (DWTs)
- dyadic DWTs (i.e., those based on a factor of 2)
- orthogonal wavelets

... From now on all things are very easy and “Hands On” ...

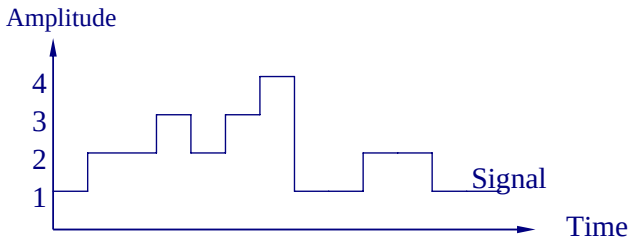
Stéphane Mallat discovered a very interesting correlation between orthogonal wavelets and filters which are much used in signal processing today.

That is the reason for the terms “high-pass filter” (~Wavelet) and “low-pass filter” (~Scaling Function).

Example: Haar Transformation (1)

We now perform a wavelet transformation with the Haar wavelet without much care about the theory.

Objective: Decomposition of a one-dimensional signal (e.g., audio) in the form of wavelet coefficients.



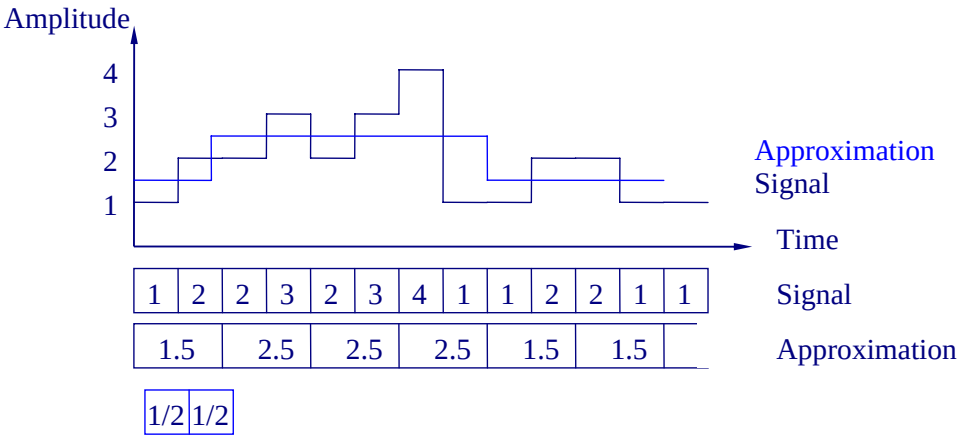
(a) Graphical Representation

1	2	2	3	2	3	4	1	1	2	2	1	1

(b) Representation by coefficients over the time

Example: Haar Transformation (2)

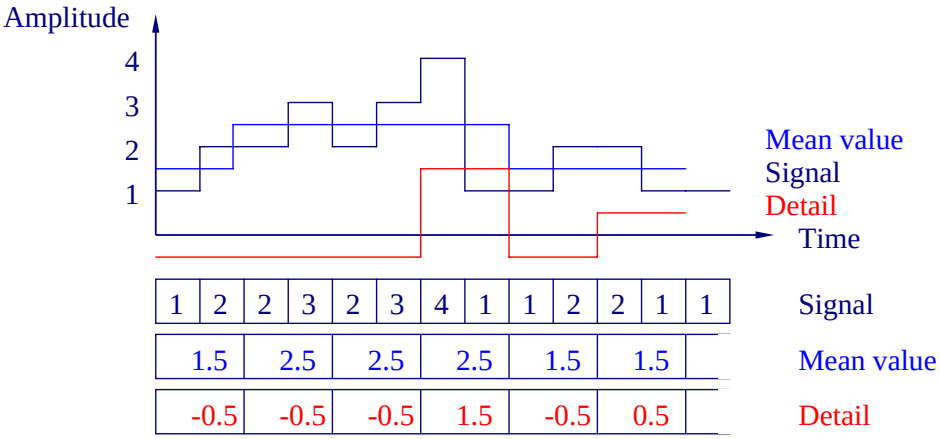
How can we represent the signal in another way without any loss of information? A rougher representation uses the mean value between two values.



Filter for the calculation of the mean value (Approximation): A filter “will be situated” over the signal. The values, which are laying “one upon the other” will be multiplicated, and all together added (→ convolution).

Example: Haar Transformation (3)

With the representation of the signal by the approximation we loose information! To reconstruct the signal we must know how far the two values are away from the mean value.



Filter for the calculation of the differences (detail):

1/2 | -1/2

Example: Haar Transformation (4)

We have transformed the original signal into another representation. Notice: The number of coefficients we need for a complete representation is unchanged. (That is the meaning of the mathematical term “base transformation”).

1	2	2	3	2	3	4	1	1	2	2	1	1
1.5		2.5		2.5		2.5		1.5		1.5		
-0.5		-0.5		-0.5		1.5		-0.5		0.5		

Signal

Mean value

Detail

To reconstruct the original signal with the approximation and the details synthesis filters are used.

1	1

Synthesis filter for the first value

1	-1

Synthesis filter for the second value

With that:

1.5*1+(-0.5)*1 = 1 (synthesis of the first value)
1.5*1+(-0.5)*(-1) = 2 (synthesis of the second value)
2.5*1+(-0.5)*1 = 2 (synthesis of the first value)
2.5*1+(-0.5)*(-1) = 3 (synthesis of the second value)
etc.

Example: Haar Transformation (5)

All together we need four filters for the decomposition and the synthesis of the original signal:

- Approximation filter for the mean value
- Detail filter for the differences
- Synthesis filter for the first value
- Synthesis filter for the second value

1/2	1/2
1/2	-1/2
1	1
1	-1

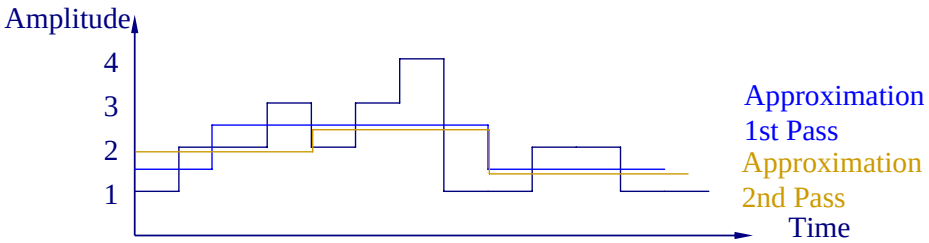
The decomposition of the signal into approximations and details can now be continued with the input signal.

Declarations:
Approximation filter := low-pass filter
Detail filter := high-pass filter

Treatment of the signal in adjustable resolutions: Multi-scale analysis.

Example: Haar Transformation (6)

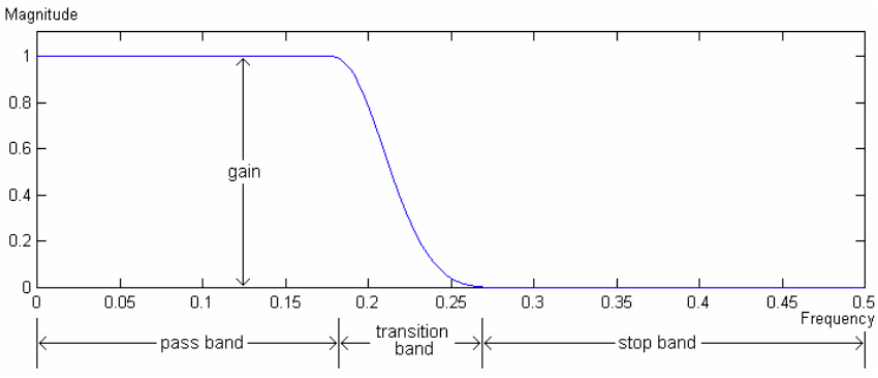
Recursion with the calculated approximation (low-pass filtered, with a “rougher” version of the input signal): Store the details (they will be needed for the synthesis), and work further with the approximations.



1	2	2	3	2	3	4	1	1	2	2	1	1
1.5		2.5		2.5		2.5		1.5		1.5		
2			2.5				1.5					
2.25												

Signal
Approx. 1
Approx. 2
Approx. 3

High- and Low-Pass Filters



The figure shows a low-pass filter. A low-pass filter lets lower frequencies pass (multiplication with 1) and blocks higher frequencies (multiplication with 0). In real filter implementations the edge is not very sharp.

A high pass filter works vice versa.

Multiresolution Analysis

If a signal (a function, a “domain”) is successively viewed on rougher scales (e.g., with the Haar Transformation), we call the process Multi-Resolution Analysis.

We look back

1	2	2	3	2	3	4	1	1	2	2	1	1
1.5		2.5		2.5		2.5		1.5		1.5		
2			2.5				1.5					
2.25												

Signal

Approx. 1

Approx. 2

Approx. 3

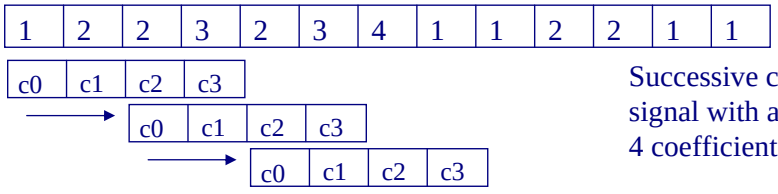
A coefficient of the signal represents a value. After the first pass through the wavelet transform the coefficient of the low-pass includes information about two signal values. After the second pass the coefficient includes information about four signal values, etc. The “scope of engagement” of a coefficient will be stretched with every step. We are doing always the same but at different resolutions.

Common Wavelet Transformation

We have learned something about the four filters of the Haar Transformation and the synthesis.

1/2	1/2
1/2	-1/2
1	1
1	-1

Wavelet filters used for image analysis and image compression are more complex. In any case, for a complete transformation, we need a low-pass filter, a high-pass filter and two synthesis filters. The filter will be placed over the signal and convoluted with it (that means a multiplication and a addition). After that, the filter will be moved about 2 elements of the signal.



Successive convoluted of the signal with a low pass filter with 4 coefficients

Important notice: With all filters with a length > 2 there exists a boundary value problem!

The Usage of Wavelets: Audio Analysis

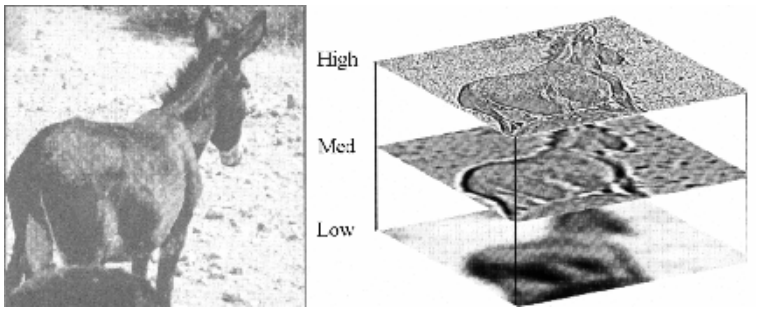
In each iteration step, the Discrete Wavelet Transformation DWT decomposes a signal into half of the resolution.

Human perception of audio signals ranges from approx. 20 Hz to 20 kHz. The main acoustic range is perceived in a logarithmic way: The frequency range from 100 Hz to 200 Hz is perceived with the same number of biological receptors as the range from 200 Hz to 400 Hz, etc.

This is just what the Wavelet Transform models!

The Usage of Wavelets: Image Compression

It is useful to represent a signal in a way similar to human perception.

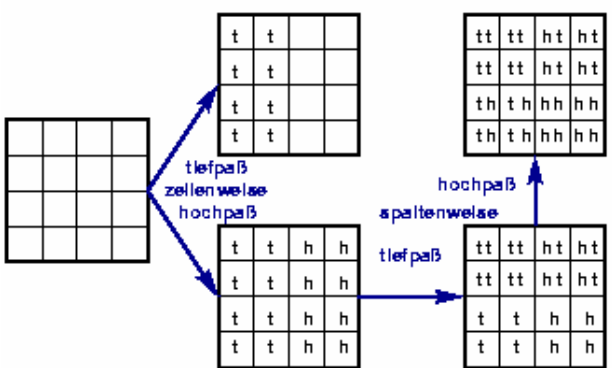


The segmentation “from rough to fine” makes it possible to start to view an image at the roughest representation. This roughest representation is supposed to contain the most important information. If the memory/bandwidth is sufficient it is possible to add more details.

Filters in Multiple Dimensions

If we use the wavelet transformation with images (2-dim) or videos (3-dim) it is necessary to extend the algorithm to multi-dimensional filters.

There is a special kind of wavelets, the so called “separable wavelets” that allow us to start with one-dimensional filters and then add more dimensions.



The figure shows a still image that is first filtered line-by-line with the high-pass and low-pass, and then column by column.

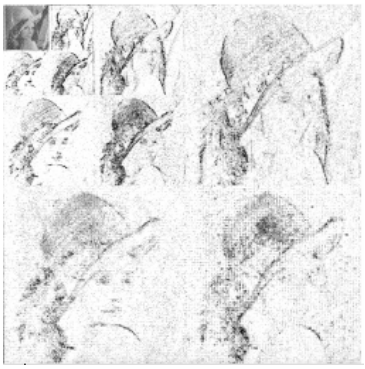
Image Compression with Wavelets (1)

Original image „Lenna“

We start first with the lines. The approximation is shown on the left, the details on the right.

The line-wise filtered image is the starting point for the column-wise filtering process. We obtain four versions in a complete recursion step.

Image Compression with Wavelets (2)



After storing the details (they are not treated any further), the approximation will now be filtered by a low-pass and a high-pass filter again. The resulting details will be stored, the approximation will be filtered again, and so on.

JPEG-2000

The new standard JPEG-2000 is based on the wavelet transformation. The visible artifacts are not as disturbing for the human as the block artifacts of DCT-based JPEG.

JPEG-2000 Example at Increasing Compression Rates (1)

JPEG-2000 Example at Increasing Compression Rates (2)