# Lecture 9: Output Data Analysis

Holger Füßler

# Course overview

| | |
|---|---|
| 1. Introduction | 7. NS-2: Fixed networks |
| 2. Building block: RNG | 8. NS-2: Wireless networks |
| 3. Building block: Generating random variates I and modeling examples | 9. Output analysis |
| 4. Building block: Generating random variates II and modeling examples | 10. Output analysis: comparing different configurations |
| 5. Algorithmics: Management of events | 11. Omnet++ / OPNET |
| 6. NS-2: Introduction | 12. Simulation lifecycle, summary |

# Part A: Output Data Analysis for a Single System

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# Structure – Part A

- Part A.I: Problem statement 'output analysis'
- Part A.II: Some probability theory and statistics
- Part A.III: Types of discrete event simulations
- Part A.IV: Credibility of simulation studies

# A.I Problem statement

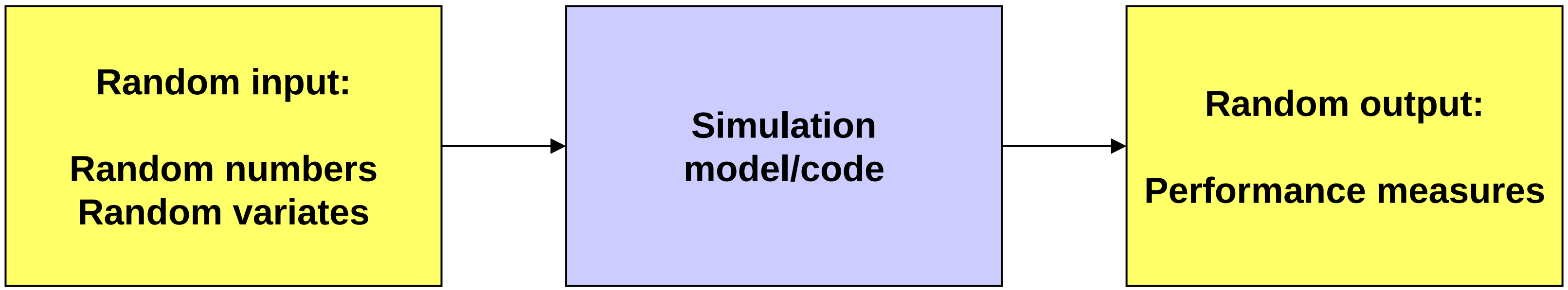
- By performing a simulation study one ‘observes’ a system.
- As output one gets a collection of data (statistics, traces).
- What ‘features’ can be inferred about the system?

**“In many simulation studies a great deal of time and money is spent on model development and ‘programming,’ but little effort is made to analyze the simulation output data appropriately.”**

**“… a simulation is a computer-based statistical sampling experiment.”**

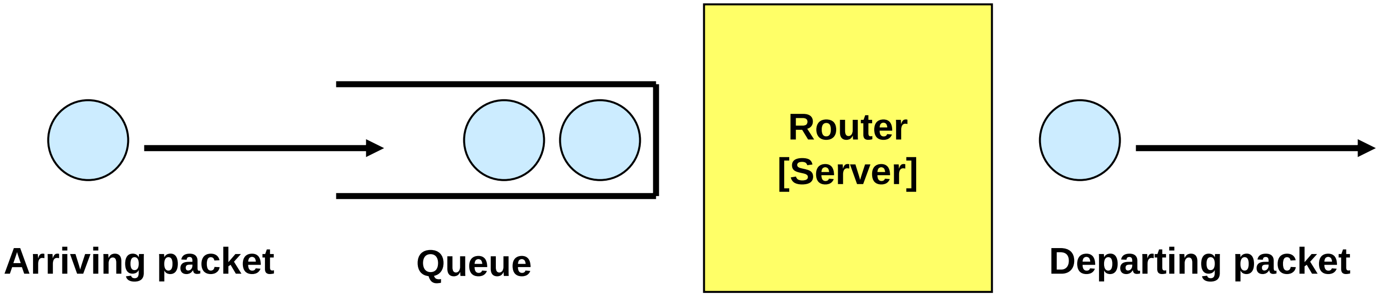
**[LK2000, Chap. 9]**

# A.I Problem statement cont‘d




- Do not get ‚exact‘ answers
- Two different runs of the same model: different numerical results

# A.I Example 1: results for M/M/1 queue




**Varying seeds (NetSim lab 3, change argument of lcgrand):**

| Replication | Average delay | Average number in queue | Server utilization |
|---|---|---|---|
| 1 | 0.606 | 0.060 | 0.192 |
| 2 | 0.554 | 0.057 | 0.206 |
| 3 | 0.586 | 0.057 | 0.200 |
| 4 | 0.452 | 0.046 | 0.197 |
| 5 | 0.490 | 0.050 | 0.199 |

# A.I Example 1: results for M/M/1 queue cont‘d

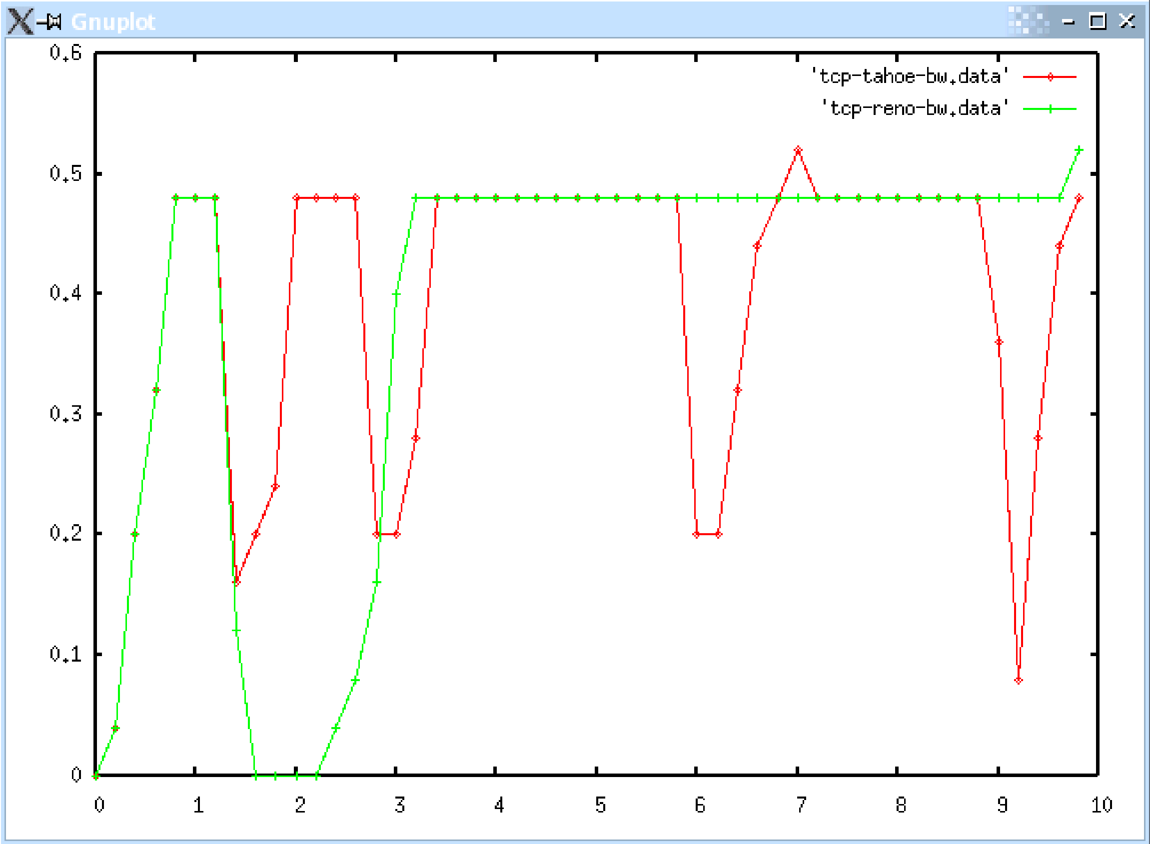
» Varying number of packets (NetSim lab 3, change parameter in mm1.in)

| Number of packets | Average delay | Average number in queue | Server utilization |
|---|---|---|---|
| 1000 | 0.606 | 0.060 | 0.192 |
| 2000 | 0.554 | 0.057 | 0.206 |
| 3000 | 0.586 | 0.057 | 0.200 |
| 4000 | 0.452 | 0.046 | 0.197 |
| 5000 | 0.490 | 0.050 | 0.199 |

» What is the ‚true‘ value?

» How much does the obtained result differ from the ‚true‘ value?

# A.I Example 2: TCP average achievable bandwidth




- **How long is the warm-up period?**
- **Assume that we have random bit/link errors: what is the achievable bandwidth?**

# A.I Example 2: TCP achievable bandwidth

```
+ 7.805824 3 2 ack 40 ------- 0 3.0 0.0 115 268
- 7.805824 3 2 ack 40 ------- 0 3.0 0.0 115 268
r 7.809248 3 2 ack 40 ------- 0 3.0 0.0 114 265
+ 7.809248 2 1 ack 40 ------- 0 3.0 0.0 114 265
- 7.809248 2 1 ack 40 ------- 0 3.0 0.0 114 265
r 7.81728 0 1 tcp 1040 ------- 0 0.0 3.0 124 266
+ 7.81728 1 2 tcp 1040 ------- 0 0.0 3.0 124 266
- 7.81728 1 2 tcp 1040 ------- 0 0.0 3.0 124 266
r 7.81744 1 2 tcp 1040 ------- 0 0.0 3.0 117 258
+ 7.81744 2 3 tcp 1040 ------- 0 0.0 3.0 117 258
- 7.81744 2 3 tcp 1040 ------- 0 0.0 3.0 117 258
r 7.818944 0 1 tcp 1040 ------- 0 0.0 3.0 125 267
+ 7.818944 1 2 tcp 1040 ------- 0 0.0 3.0 125 267
r 7.822464 2 3 tcp 1040 ------- 0 0.0 3.0 116 257
+ 7.822464 3 2 ack 40 ------- 0 3.0 0.0 116 269
- 7.822464 3 2 ack 40 ------- 0 3.0 0.0 116 269
r 7.825888 3 2 ack 40 ------- 0 3.0 0.0 115 268
…
```

» **Conversion to meaningful results?**

# A.I General set-up

m Observations

n Experiments

$$
\begin{matrix}
Y_{11} & \dots & Y_{1i} & \dots & Y_{1m} \\
Y_{21} & \dots & Y_{2i} & \dots & Y_{2m} \\
\dots & \dots & \dots & \dots & \dots \\
Y_{n1} & \dots & Y_{ni} & \dots & Y_{nm}
\end{matrix}
$$

Warm-up period

**Challenge:**
**Choose n, m and warm-up period approriately!**

# A.II Estimates of means and variances

» Suppose that $X_1, X_2, \ldots, X_n$ are IID random variables with finite mean $\mu$ and finite variance $\sigma^2$.

» How can we get an estimate for $\mu$ and $\sigma$?

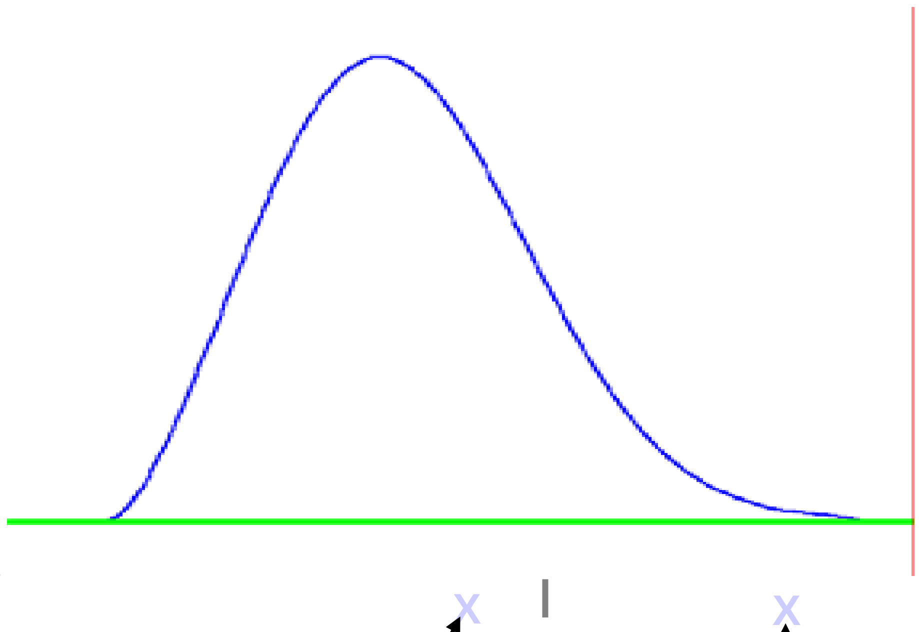
» Unbiased estimator for $\mu$: sample mean

$$\bar{X}(n) = \frac{\sum_{i=1}^{n} X_i}{n}$$

» Unbiased estimator for $\sigma^2$: sample variance

$$S^2(n) = \frac{\sum_{i=1}^{n} [X_i - \bar{X}(n)]^2}{n-1}$$

Density function for $\bar{X}(n)$




First observation of $\bar{X}(n)$

Second observation of $\bar{X}(n)$

# A.II Estimates of means and variances (Calculation)

let $X_j^{\Sigma}$ be the sum of all values until the $j$-th element and $X_j^{\Sigma^2}$ the sum of the respective squares, i.e.

$$X_j^{\Sigma} = \sum_{i=1}^{j} X_i \qquad X_j^{\Sigma^2} = \sum_{i=1}^{j} X_i^2$$

then, the estimators can be calculated as

$$\bar{X}(n) = \frac{X_n^{\Sigma}}{n}$$

$$S^2(n) = \frac{1}{n-1}\left(X_n^{\Sigma^2} - 2 \cdot (\bar{X}(n)) \cdot X_n^{\Sigma} + n \cdot (\bar{X}(n))^2\right)$$

→ **we only need the accumulated sum / squared sum of each variable**

# A.II Confidence intervals for the mean

- Again, assume $X_1$, $X_2$, …, $X_n$ are IID random variables with finite mean $\mu$ and finite variance $\sigma^2$ greater 0.
- Central limit theorem states: $\bar{X}$(n) is approximately distributed as a normal random variable with mean $\mu$ and variance $\sigma^2$/n.
- For sufficiently large n, an approximate 100(1-$\alpha$) percent confidence interval for $\mu$ is given by

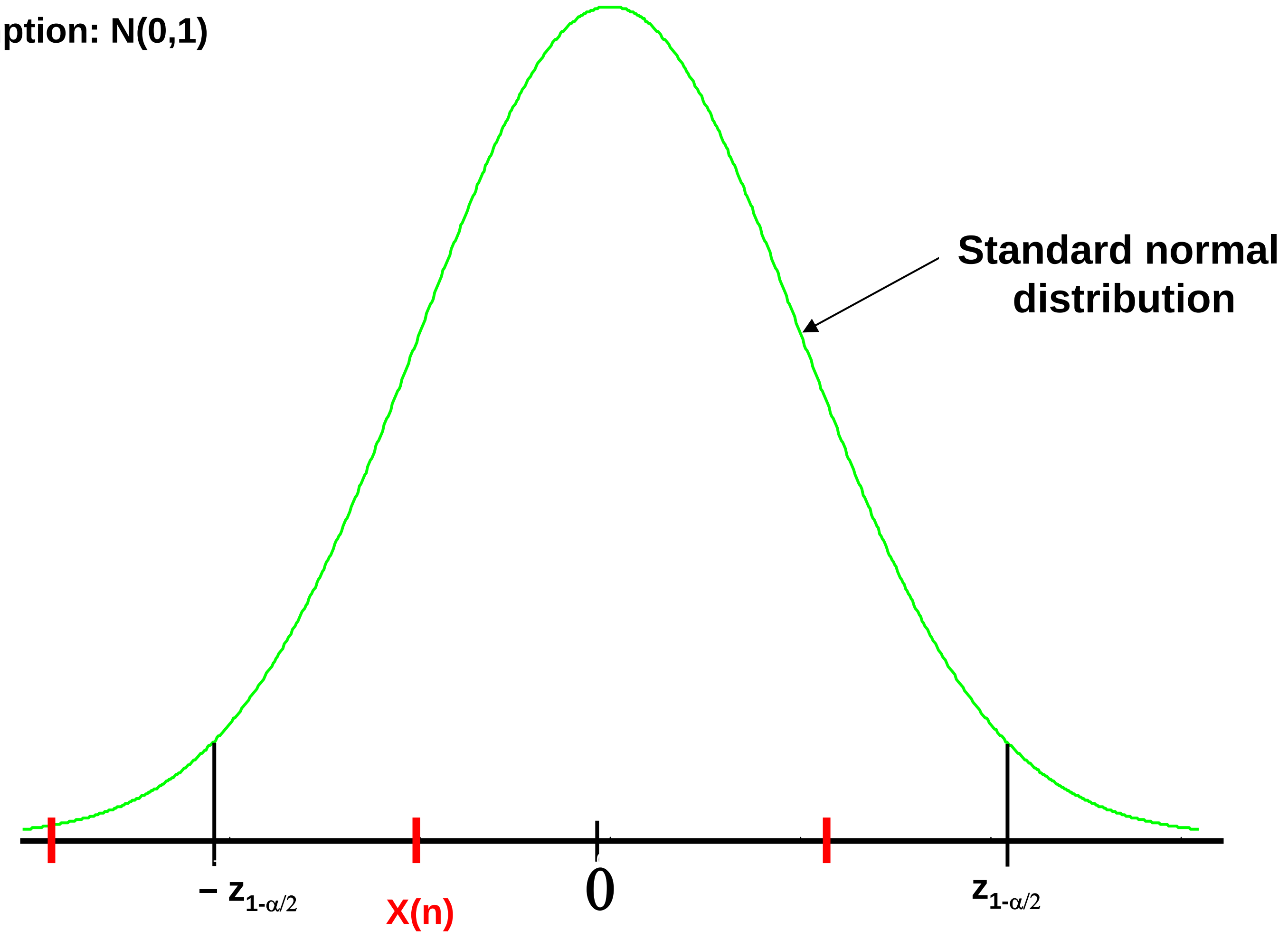
$$\bar{X}(n) \pm z_{1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}$$

- Interpretation of 'confidence interval': "… in 100(1-$\alpha$) percent of all cases the true parameter $\mu$ is within the interval.
- Why 'approximate'?
  - It is only asymptotically correct

**Warning: Only „good" for sample size of approx. > 50**

# A.II Confidence intervals for the mean: illustration

**Assumption: N(0,1)**



# A.II Example: M/M/1 queue

| Replication | Average delay |
|---|---|
| 1 | 0.606 |
| 2 | 0.554 |
| 3 | 0.586 |
| 4 | 0.452 |
| 5 | 0.490 |
| 6 | 0.548 |
| 7 | 0.519 |
| 8 | 0.498 |
| 9 | 0.366 |
| 10 | 0.364 |
| Average (.3digits) | 0.498 |

- **Experiments for 1000 packets each**
- **$S^2(10) = 0.007$**
- **95% confidence interval:**

  **$z \approx 2.0$**
- **$0.498 \pm 0.014$**

# A.II Selecting the sample size

- Result so far (under a lot of assumptions): for sufficiently large n, an approximate 100(1-$\alpha$) percent confidence interval for $\mu$ is given by

$$\bar{X}(n) \pm \underbrace{z_{1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}}$$

How do we have to select the number of samples n?

Let us bound this by an absolute value v

- Thus, given $S^2(n)$ (or $\sigma^2$ if it is known) and value v and $z_{1-\alpha/2}$, one can solve for number of samples n.

# A.II Example: calculating required sample sizes

- Let $Y_i$, i=1, 2, …, be IID Bernoulli random variables with parameter p.
- What is the sample size necessary to estimate p within 0.05 with probability .95?
- Assume no information is given w.r.t. the variance.
- Max. variance: 0.025
- $0.025 \cdot (2/0.05)^2 = n$
- n=400

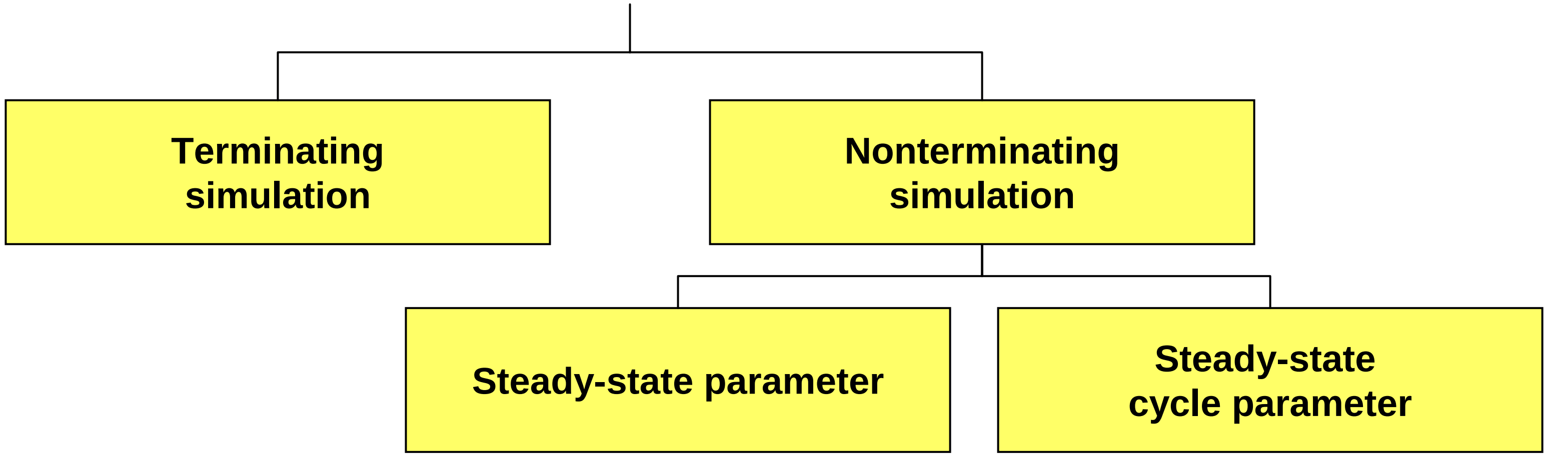
# A.II Experiment: Estimated coverages

- Coverage: proportion of confidence intervals that contain the ‚true' parameter $\mu$.
- Should be 1 - $\alpha$ for ‚n sufficiently large'
- Can be checked for known distributions.

| Distribution | Skewness | M=5 | M=10 | M=20 | M=40 |
|---|---|---|---|---|---|
| Normal | 0.00 | 0.910 | 0.902 | 0.898 | 0.900 |
| Exponential | 2.00 | 0.854 | 0.878 | 0.870 | 0.890 |
| Chi square | 2.83 | 0.810 | 0.830 | 0.848 | 0.890 |
| Lognormal | 6.18 | 0.758 | 0.768 | 0.842 | 0.852 |
| Hyperexp. | 6.43 | 0.584 | 0.586 | 0.682 | 0.774 |

**Estimated coverages for 90 percent confidence intervals based on 500 independent experiments for each of the sample sizes [Source: Law/Kelton]**

# A.III Types of simulations w.r.t. output analysis




*Terminating*: Parameters to be estimated are defined relative to specific initial and stopping conditions that are part of the model

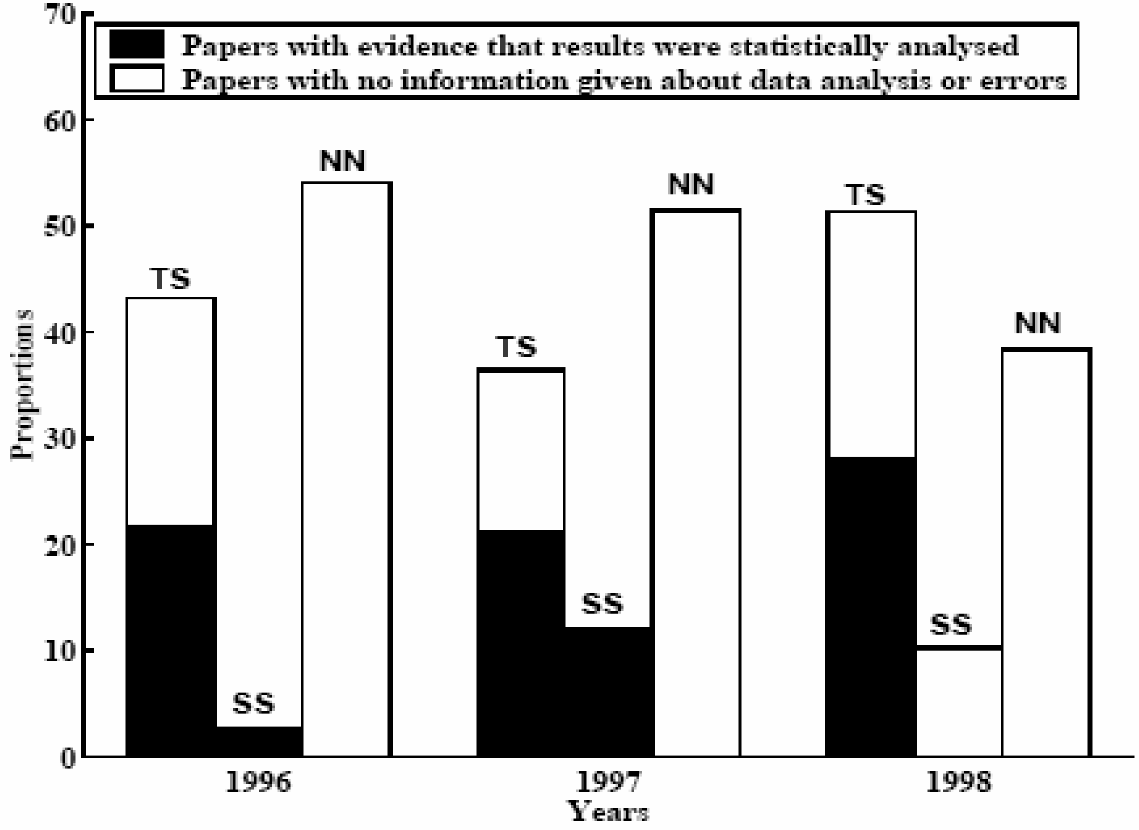
*Nonterminating*: There is no natural and realistic event that terminates the model
Interested in "long-run" behavior characteristic of "normal" operation
If the performance measure of interest is a characteristic of a steady-state distribution of the process, it is a *steady-state* parameter of the model

Not all nonterminating systems are steady-state: there could be a periodic "cycle" in the long run, giving rise to *steady-state cycle* parameters

# A.III Types of simulations w.r.t. output analysis

- Terminating simulations: „9 to 5 scenarios“
- Example: M/M/1 queue
  - Initial condition: empty queue
  - Terminating condition: time elapsed
- Statistics for terminating simulations: see Part II of this lecture
- Challenge: steady-state simulations
  - How to get rid of impact of initial condition?
  - When to stop simulation?

# A.IV „Crisis of credibility“

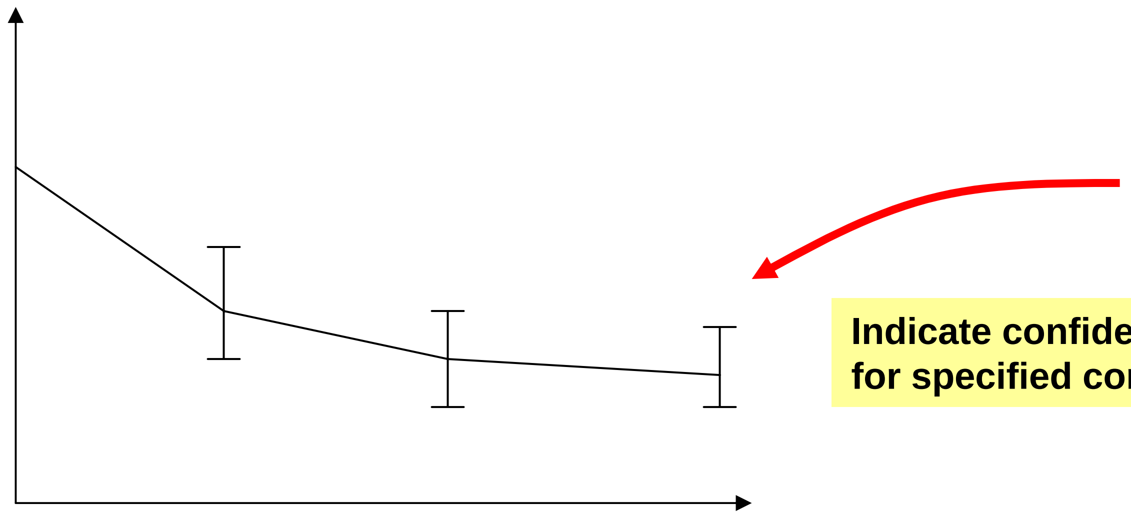



(c) IEEE/ACM Transactions on Networking

K. Pawlikowski, H.-D. Joshua Jeong, and J.-S. Ruth Lee, ***On credibility of simulation studies of telecommunication networks***, **IEEE Communications Magazine 40(1).**

# A.IV Recommendations (Pawlikowski et al.)

- Reported simulation experiments should be repeatable
  - Give information about
    - The PRNG(s) used during the simulation
    - The type of simulation
    - The method of analysis of simulation output data
    - The final statistical errors associated with the results



Indicate confidence interval (CI) for specified confidence level (CL)

# A.IV Comments for ‚best practices‘

- **Independence or covariance-stationarity rarely encountered in practice ☺**
- **But: if the number of replications, samples etc. is too low even under the assumptions of independence or covariance-stationarity, something is probably flawed …**
  - **We need mathematical results to check**
- **In reality, also time and space constraints can severely impact achievable confidence intervals**
  - **But this should be specified**
- **Trace and plot as many variables as possible to cross-check correctness ☺**

# A.IV Wrap-up

- **Any stochastic computer simulation (using RNGs/(PRNGs) has to be regarded as a (simulated) statistical experiment.**
- **Statistics background:**
  - **estimating means and variances**
  - **confidence levels and intervals**
  - **hypothesis testing**
- **Transient and steady-state behavior**
- **Terminating, steady-state and cyclic steady state simulations**
- **The issue of credibility**

# References – Part A

- Averill M. Law, W. David Kelton: "Simulation Modeling and Analysis", McGraw-Hill, 3rd edition, 2000
  - Chapters 4 and 9

# Part B: Comparing Different Configurations

# Where we are …

**Why network simulations?**

- **Educational use**
  - **See protocol in action**
  - **Does it work as intended?**
- **Get some quantitative results for a single configuration**
  - **E.g., how long does it take to find a route?**
- **Compare different configuration and decide which one is ‘better’**
  - **Trade-offs**

**Design phase**

**Tuning phase**

**Decision phase**

**Getting harder**

# Lecture overview – Part B

- **Part B.I: Scope of this Lecture - Motivation**
- **Part B.II: Comparison of different configurations**
- **Part B.III: Variance reduction techniques**

## B.I Comparing different configurations: e.g. Different Routing Protocols

» **Which one is ‘better’?**

- **Answer depends on metric(s)**
- **Answer depends on scenario**
  - **Mobility pattern**
  - **Communication pattern**
  - **Caching strategies**
  - **Flooding strategies**
  - **…**

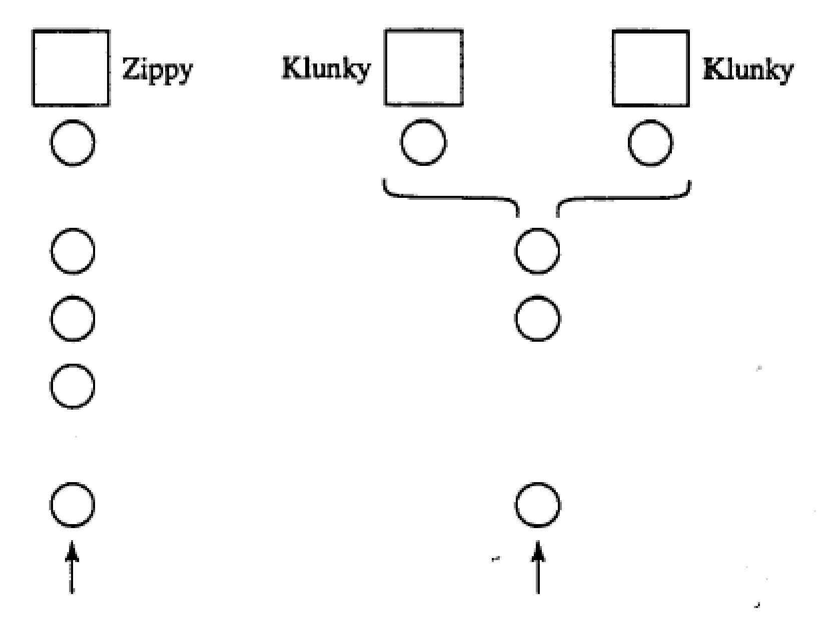
» **Metrics:**

- **Packet delivery ratio**
- **Route acquisition time**
- **End-to-end delay**
- **Overhead costs**

# B.II Comparison of two different configurations

- Let's go back to some simple scenario

- Compare M/M/1 queue (service time: exponential with mean 0.9) with M/M/2 queue (service time: exponential with mean 1.8 each)

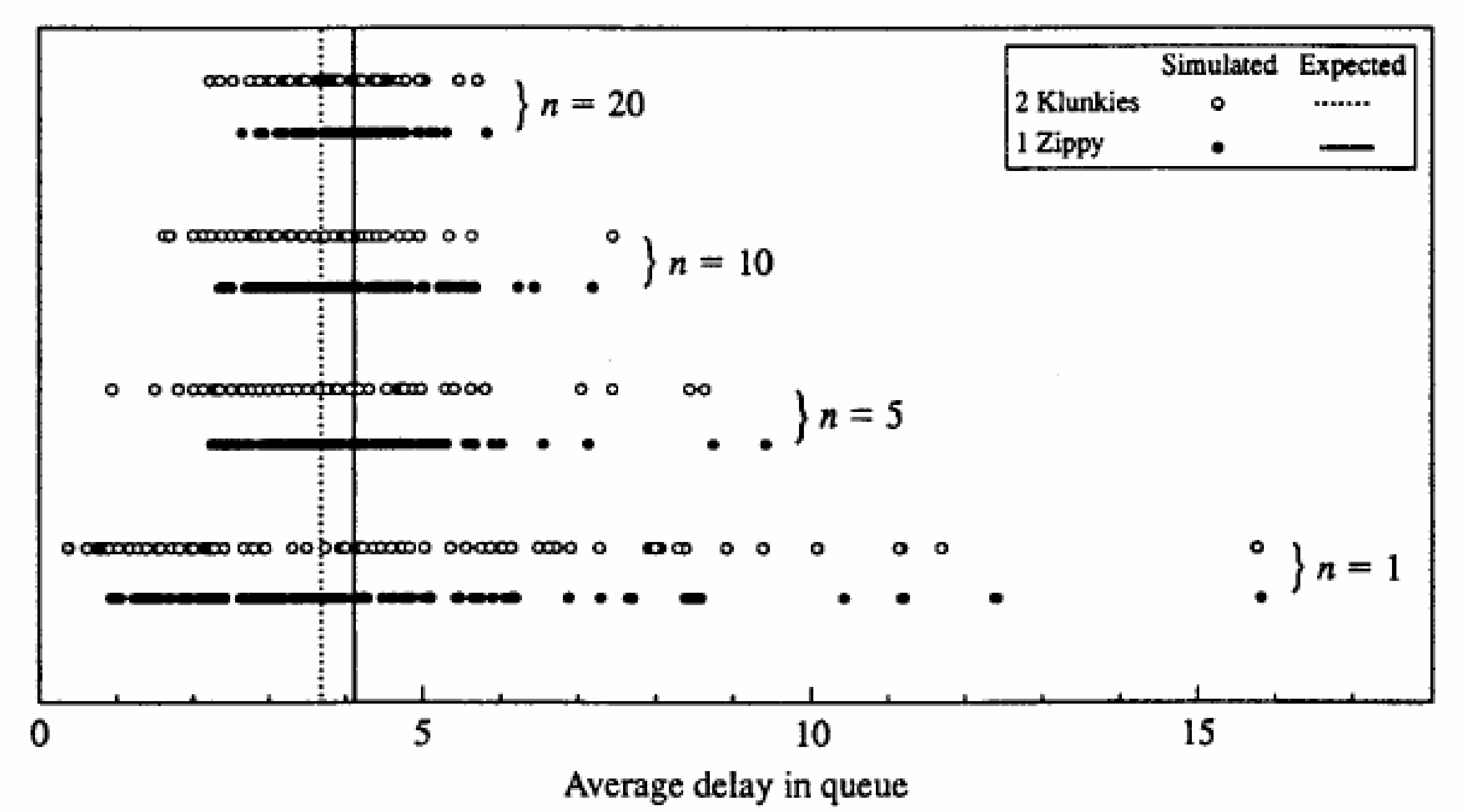


# B.II Motivation by example 2

| Experiment | $\hat{d}_Z(100)$ | $\hat{d}_K(100)$ | Recommendation | |
|---|---|---|---|---|
| 1 | 3.80 | 4.60 | Zippy | (wrong) |
| 2 | 3.17 | 8.37 | Zippy | (wrong) |
| 3 | 3.96 | 4.18 | Zippy | (wrong) |
| 4 | 1.91 | 5.77 | Zippy | (wrong) |
| 5 | 1.71 | 2.23 | Zippy | (wrong) |
| 6 | 6.16 | 4.72 | Klunky | (right) |
| 7 | 5.67 | 1.39 | Klunky | (right) |
| ⋮ | ⋮ | ⋮ | ⋮ | |
| 98 | 8.40 | 9.39 | Zippy | (wrong) |
| 99 | 7.70 | 1.54 | Klunky | (right) |
| 100 | 4.64 | 1.17 | Klunky | (right) |

| $n$ | $P$(wrong answer) |
|---|---|
| 1 | 0.52 |
| 5 | 0.43 |
| 10 | 0.38 |
| 20 | 0.34 |

# B.II Motivation by example 3



# B.II Confidence interval for the difference between two systems

- **Two alternative simulated systems (*i* = 1, 2), $\mu_i$ = expected performance measure from system *i***
- **Take "sample" of $n_i$ observations (replications) from system *i***
- **$X_{ij}$ = observation *j* from system *i***
- **Want: confidence interval on $z = \mu_1 - \mu_2$**
- **If interval misses 0, conclude there is a *statistical* difference between the systems**
- **Is the difference *practically* significant? Must use judgment in context.**

# B.II Paired confidence interval

- Assume $n_1 = n_2$ (=n, say)
- For a fixed j, $X_{1j}$ and $X_{2j}$ need not be independent
  - Important for variance reduction techniques (next part)
- Let $Z_j = X_{1j} – X_{2j}$
- Problem reduced to 'single system problem'
- Find confidence interval for $E[Z_j]$
- Previous example (10 runs):

$$\bar{Z}(10) = 0.376$$
$$S^2(10)/90 \approx 1.25$$
$$\text{Confidence interval for CL 95\%: } 0.376 \pm 2.24$$

# B.II Issues not covered in this part

- Other comparison methods
- Comparing more than two systems
- Ranking and selection

# B.III Variance reduction techniques

» **Main drawback of using simulation to study stochastic models:**

**Results are uncertain — have *variance* associated with them**

» **Would like to have as little variance as possible — more precise results**

» **One sure way to decrease the variance:**

**Run it some more (longer runs, additional replications)**

» **Sometimes can manipulate simulation to reduce the variance of the output at little or no additional cost — *not* just by running it some more**

» **Another way of looking at it — try to achieve a desired level of precision (e.g., confidence-interval smallness) with less simulating — *Variance-reduction technique* (VRT)**

# B.III Common random numbers (CRN)

- When comparing two or more alternative system configurations
- Basic idea: compare alternative configurations ‘under similar experimental conditions’ – use random numbers ‘for same purpose’
  - Often used ‘unconsciously’
  - E.g. use same movement and communication pattern when comparing two ad-hoc routing protocols
- Example of ‘what can go wrong’:

| $k$ | $U_k$ | Usage in $M/M/1$ | Usage in $M/M/2$ | Agree? |
|---|---|---|---|---|
| 1 | 0.401 | A | A | Yes |
| 2 | 0.614 | A | A | Yes |
| 3 | 0.434 | S | S | Yes |
| 4 | 0.383 | A | A | Yes |
| 5 | 0.506 | S | S | Yes |
| 6 | 0.709 | A | A | Yes |
| 7 | 0.185 | S | S | Yes |
| 8 | 0.834 | A | A | Yes |
| 9 | 0.646 | A | S | No |
| 10 | 0.376 | A | A | Yes |

# B.III Mathematical basis for CRN

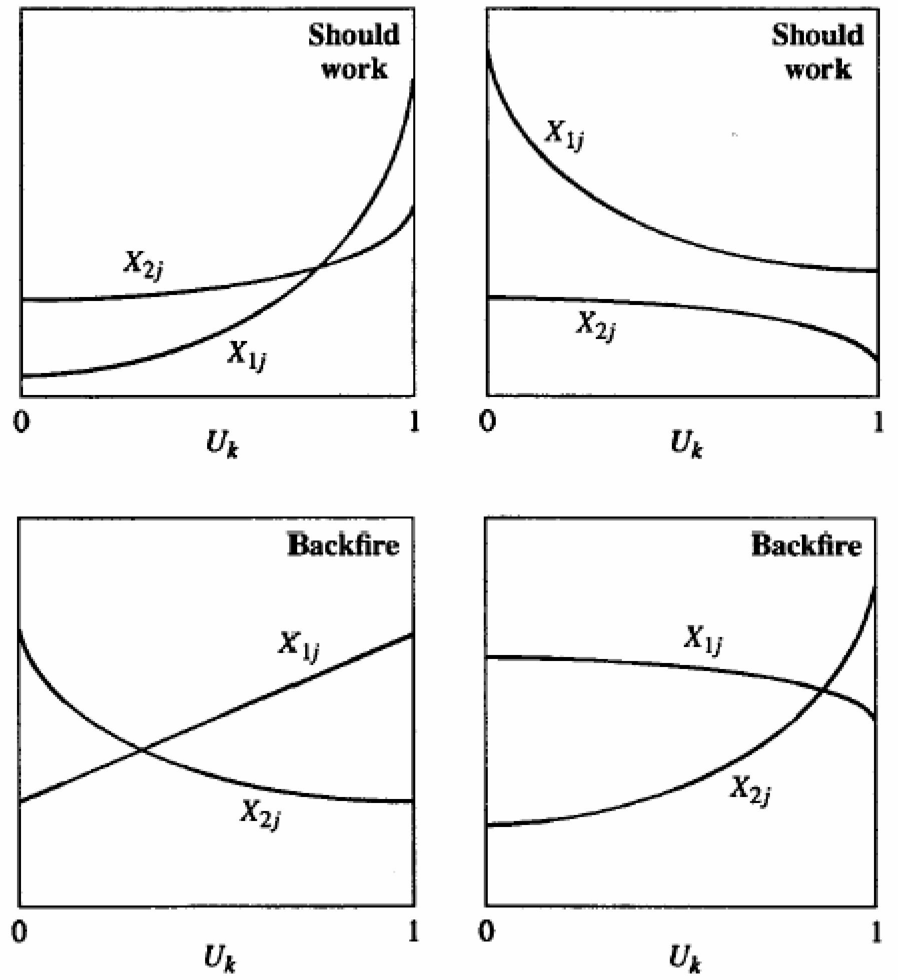
We have two alternatives, where $X_{1j}$ and $X_{2j}$ are the observations from the first and second configuration on the jth independent replication.

Again, let $Z_j = X_{1j} - X_{2j}$.

$$Var[\bar{Z}(n)] = \frac{Var(Z_j)}{n} = \frac{Var(X_{1j}) + Var(X_{2j}) - 2Cov(X_{1j}, X_{2j})}{n}$$

**When we can induce some positive correlation, we can make ... smaller.**

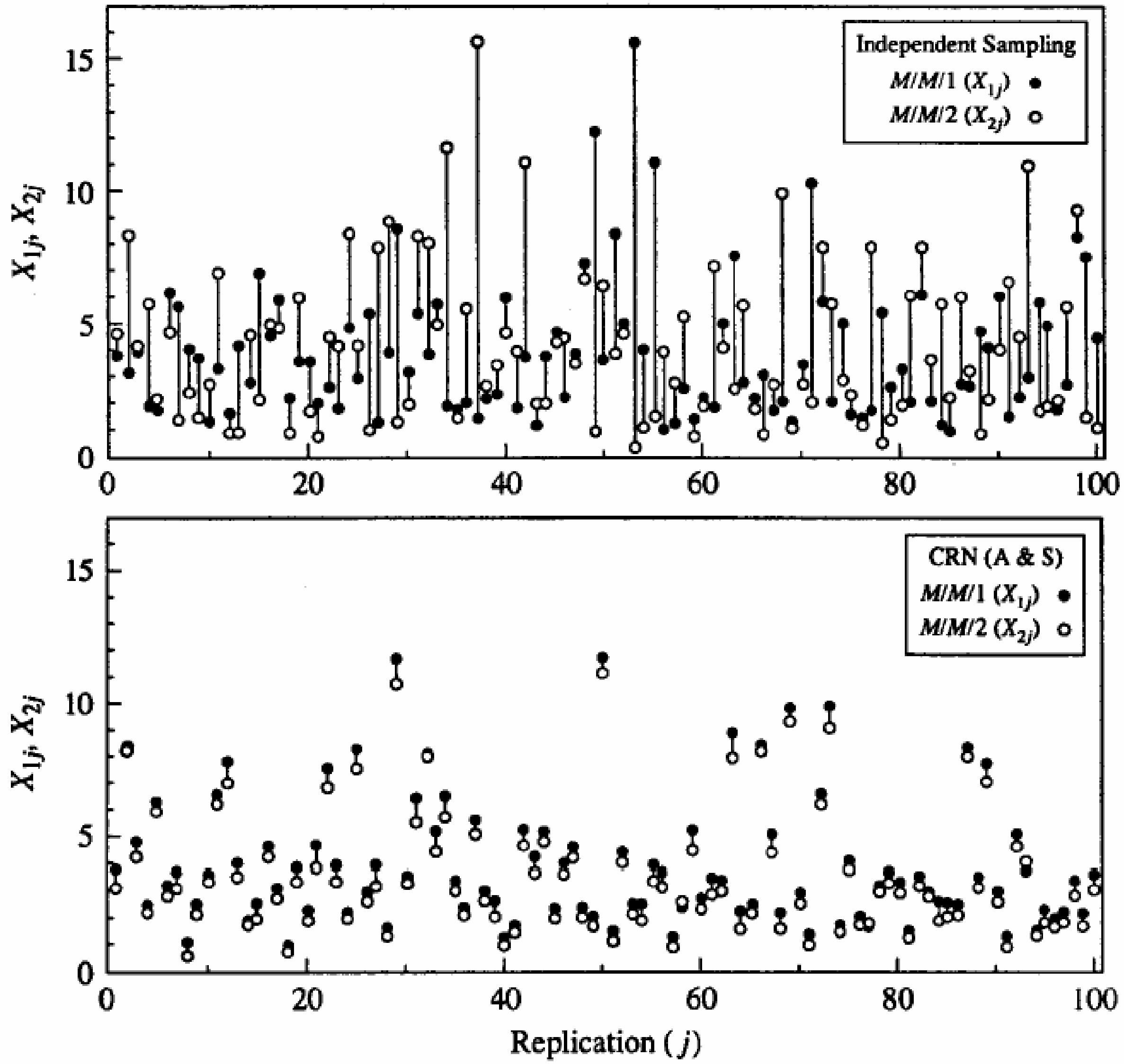
# B.III Applicability of CRN



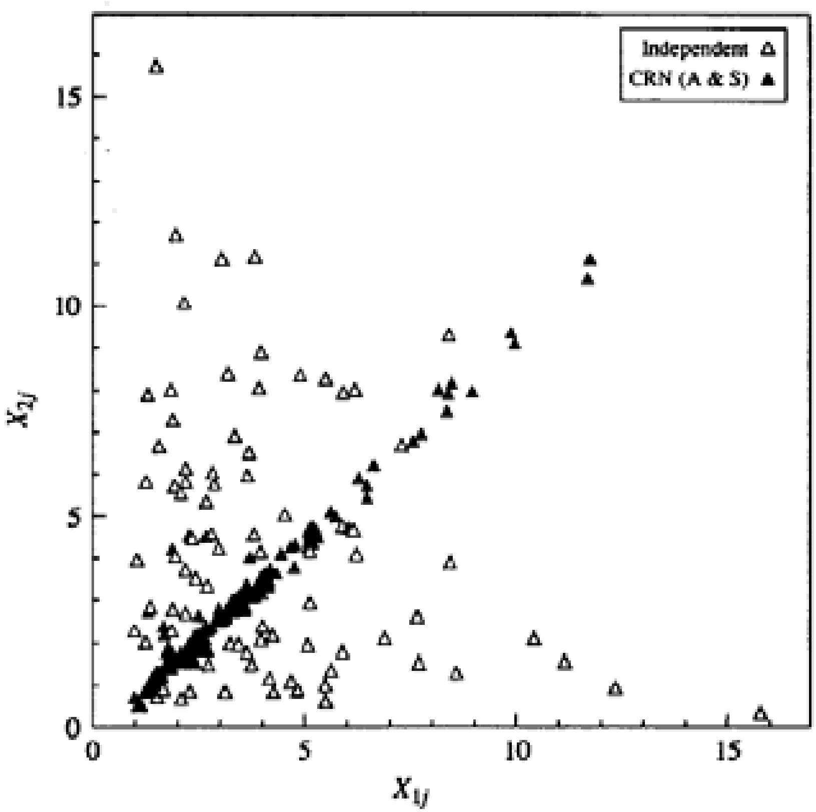
# B.III CRN – Synchronization techniques

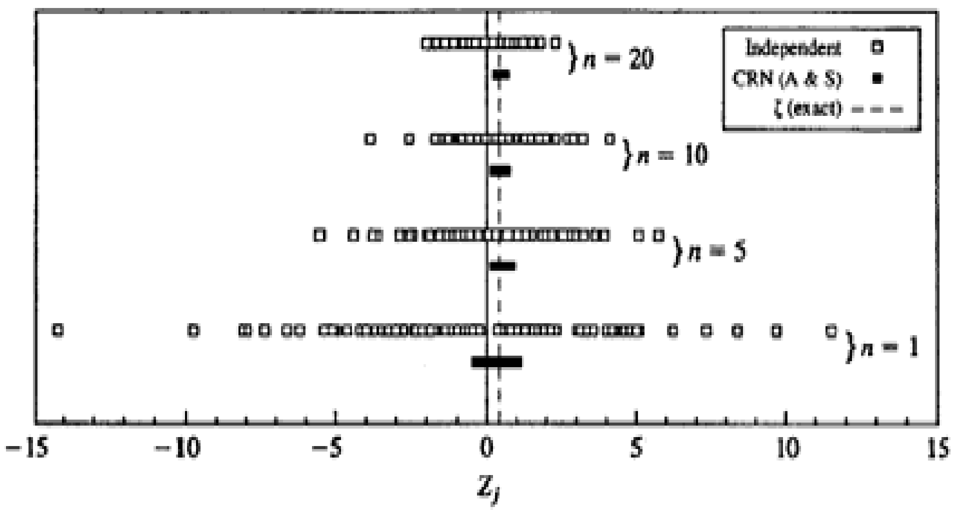
- Use of dedicated random number streams
- Use of inverse transform
  - But: inverse transform not always the most efficient choice …
- Compute random numbers in advance
  - Costs some memory
- … or waste some random numbers
  - … to keep things synchronized

# B.III CRN at work 1



# B.III CRN at work 2



# B.IV Application to network simulations

- When comparing two alternatives
  - Use ‘same’ topology
    - E.g. preprocessed movement pattern, same radio transmission range
    - E.g. same (preprocessed) link error patterns
  - Use ‘same’ communication pattern
- What else is ‘random’ and can affect results?

# Wrap-up Part B

- Today's focus: comparison of two alternative configurations
- Problem reduced to finding a confidence interval of a 'single' system
- Confidence intervals: computation for specified precision
- Precision corresponds to variance (of sample mean): variance reduction techniques needed
  - Common random numbers

# References – Part B

» **Law, Kelton: Chapters 10 and 11**