Generating Random Variates II and Examples

Holger Füßler

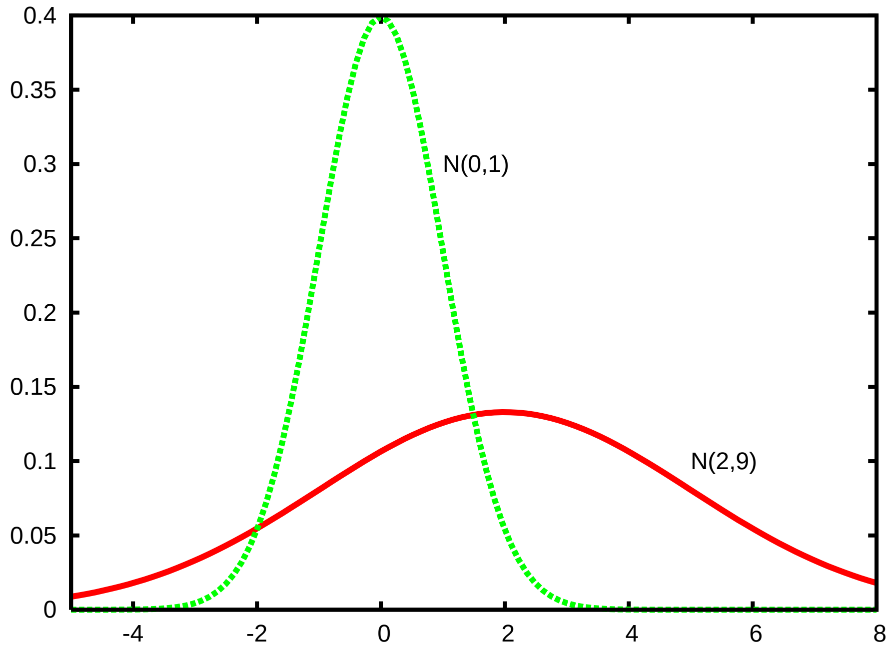
Side note: TexPoint

- **TexPoint is a Powerpoint add-in that enables the easy use of Latex symbols and formulas in Powerpoint presentations.**
- **There are two main modes of operation: inline and display.**
- **In inline mode you can use Latex symbol-macro invocations such as "\alpha^2 \times \beta_0" on your Powepoint slides.**
- **In the display mode you can write any Latex source and Latex is run to produce a bitmap that is then inserted on the slide. The bitmap remembers its Latex source so you can modify it later.**
- **http://raw.cs.berkeley.edu/texpoint/index.html**

Course overview

1. Introduction	7. NS-2: Fixed networks
2. Building block: RNG	8. NS-2: Wireless networks
3. Building block: Generating random variates I and modeling examples	9. Output analysis: single system
4. Building block: Generating random variates II and modeling examples	10. Output analysis: comparing different configuration
5. Algorithmics: Management of events	11. Omnet++ / OPNET
6. NS-2: Introduction	12. Simulation lifecycle, summary

I Generation of normal variates



Density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

μ mean

σ standard deviation

Recommendation: Play with Gnuplot!

Given X ~ N(0,1) we can obtain X' ~ N(μ, σ^2) by setting X' = μ + σ X (see next slide).

Thus, we focus on N(0,1).

I Applications

- **Citing Law/Kelton:**

  "Errors of various types, e.g., in the impact point of a bomb;

  quantities that are the sum of large number of other quantities

  (by virtue of central limit theorem)."

- **Central limit theorem: let X_1, X_2, … be IID random variables with mean μ and variance $\sigma^2 < \infty$:**

$$T_n := \frac{\sum_{j=1}^{n} X_j - n\mu}{\sigma\sqrt{n}} \approx N(0,1)$$

I Recap: general process of generating random variates

» **Formal algorithm - depends on desired distribution.**

» **But *all* algorithms have the same general form:**

- **Generate one or more IID U(0, 1) random numbers**
- **Transformation (depends on desired distribution)**
- **Return *X* ~ desired distribution**

I Code example (NS-2)

tools/rng.cc:

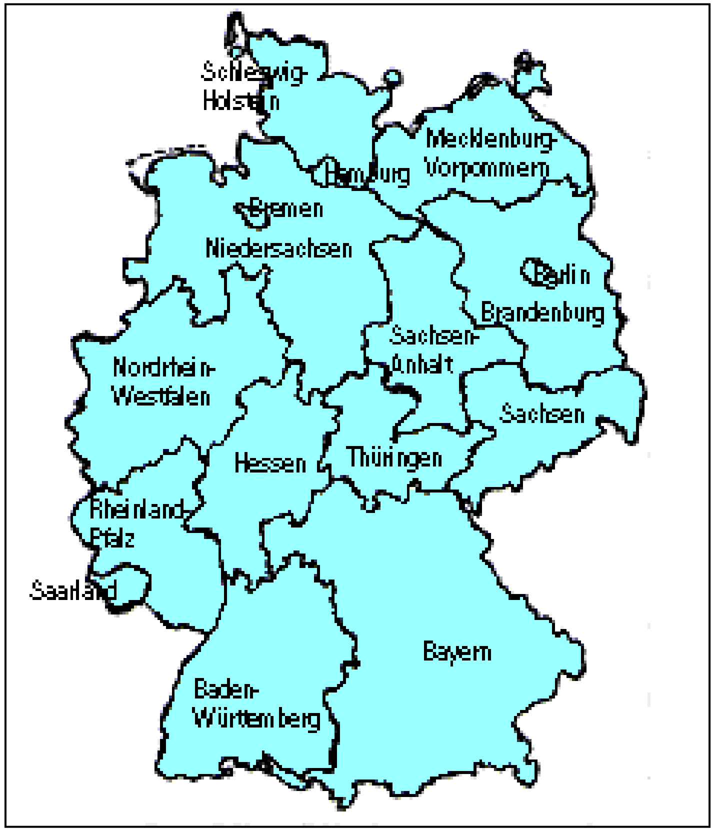
- Generate random numbers U_1 and U_2
- Set
  - $V_1 = 2\ U_1 - 1$
  - $V_2 = 2\ U_2 - 1$
  - $R^2 = V_1^2 + V_2^2$
- If $R^2 > 1$ return to step 1
- Return indep. normals

```
double
RNG::normal(double avg, double std)
{
        static int parity = 0;
        static double nextresult;
        double sam1, sam2, rad;

        if (std == 0) return avg;
        if (parity == 0) {
                sam1 = 2*uniform() - 1;
                sam2 = 2*uniform() - 1;
                while ((rad = sam1*sam1 + sam2*sam2) >= 1) {
                        sam1 = 2*uniform() - 1;
                        sam2 = 2*uniform() - 1;
                }
                rad = sqrt((-2*log(rad))/rad);
                nextresult = sam2 * rad;
                parity = 1;
                return (sam1 * rad * std + avg);
        }
        else {
                parity = 0;
                return (nextresult * std + avg);
        }
}
```

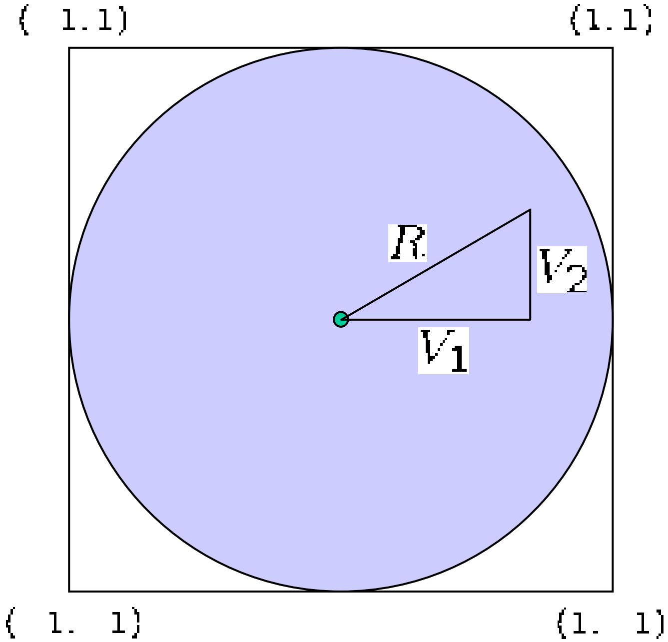
rad==0?

III Acceptance-rejection method: introduction



» **How to generate a uniform distribution for a (bounded) irregular shape?**

III Acceptance-rejection method: introduction



Start of algorithm:

- **Generate random numbers U1 and U2**
- **Set**
  - $V_1 = 2\,U_1 - 1$
  - $V_2 = 2\,U_2 - 1$
  - $R^2 = V_1^2 + V_2^2$
- **If $R^2 > 1$ return to step 1**

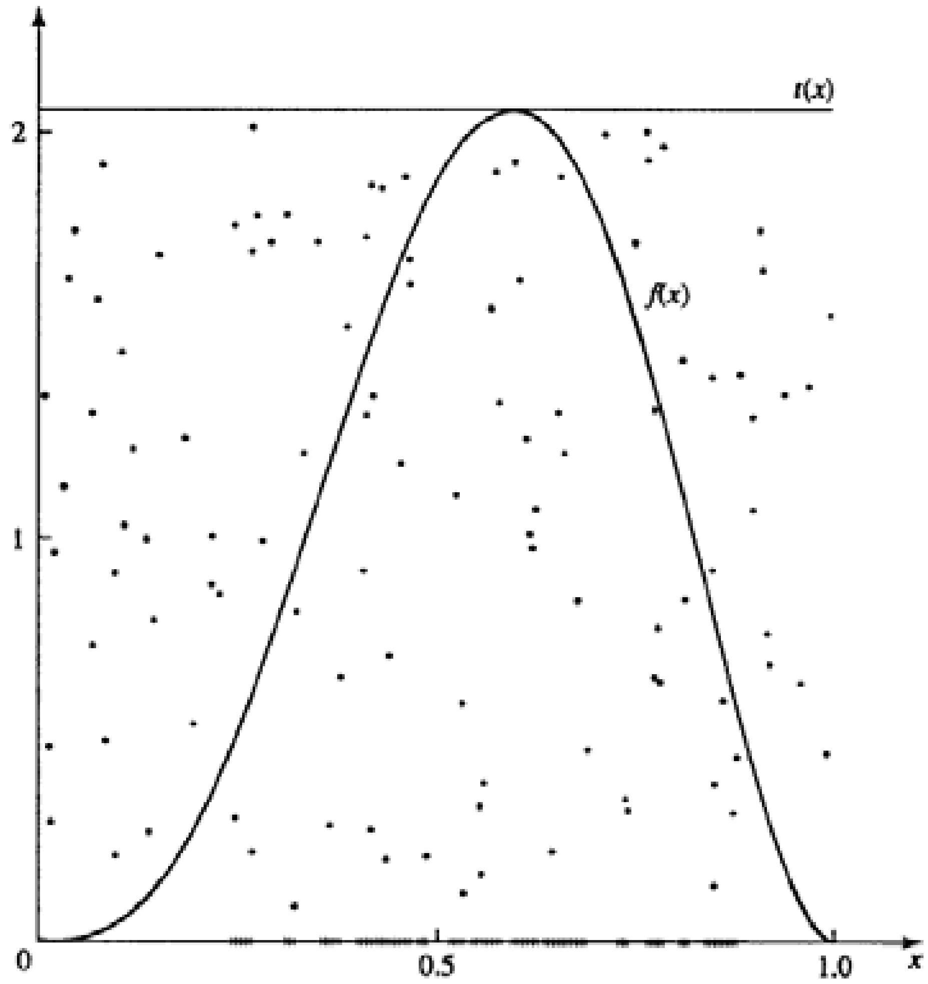
1. Select a point in the ‘bounding box’ by sampling from a uniform distribution.

2. Check, if the point is in the shaded area:

1. **If not, go to step 1.**
2. **If yes, select it as output value.**

Generates uniform distribution on shaded area.

III Acceptance-rejection method:



$f(x) = 60\,x^3\,(1-x)^2$ for $0 \le x \le 1$

» **Do same thing as before, but take projection to x-axis as output value.**

III Acceptance-rejection method: general algorithm

Goal: generate random variate

X with density function f(x).

Specify function t(x) such that t(x) $\geq$ f(x)

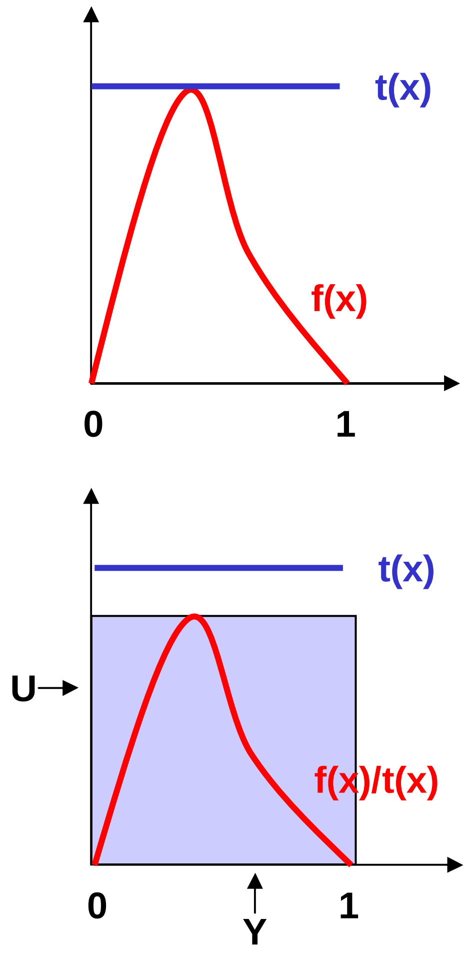
for all x such that $\int$ t(x) = c < ∞.

Define r(x) = t(x)/c; it's a density.

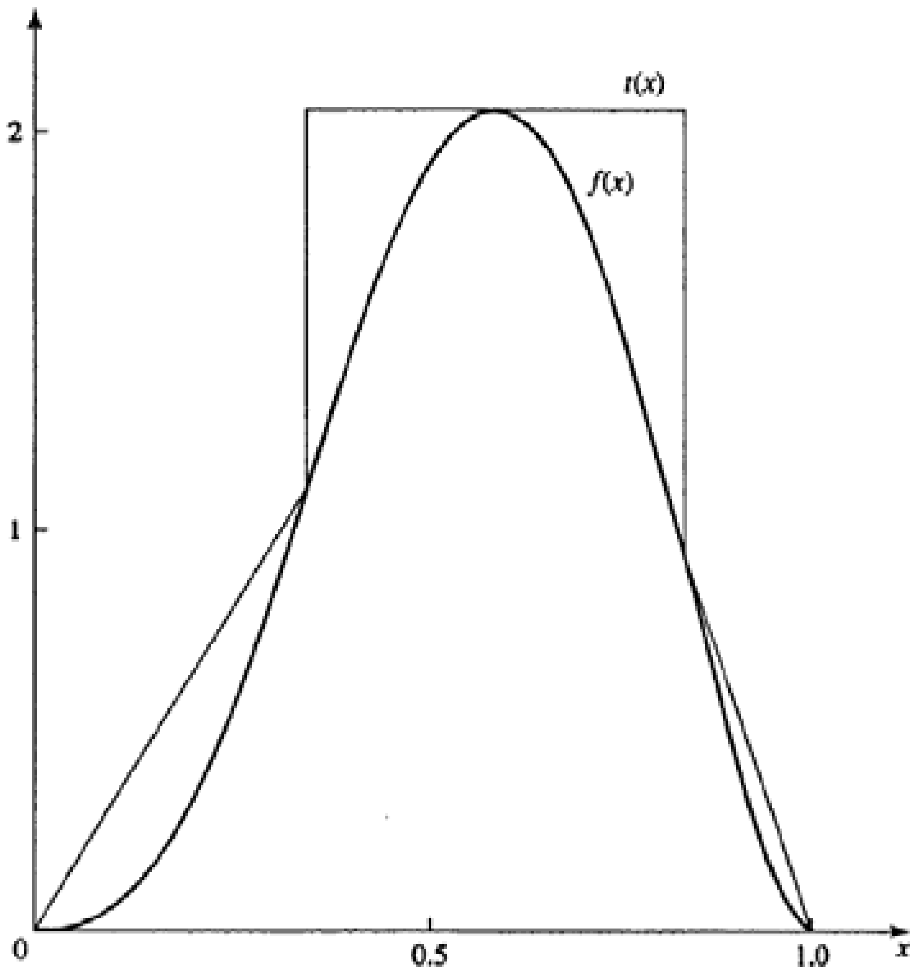
1. **Generate variate Y with density r(x).**
2. **Generate U~U(0,1) independent of Y.**
3. **If U $\leq$ f(Y)/t(Y), return X=Y and stop;**

   else go back to step 1.

Example:



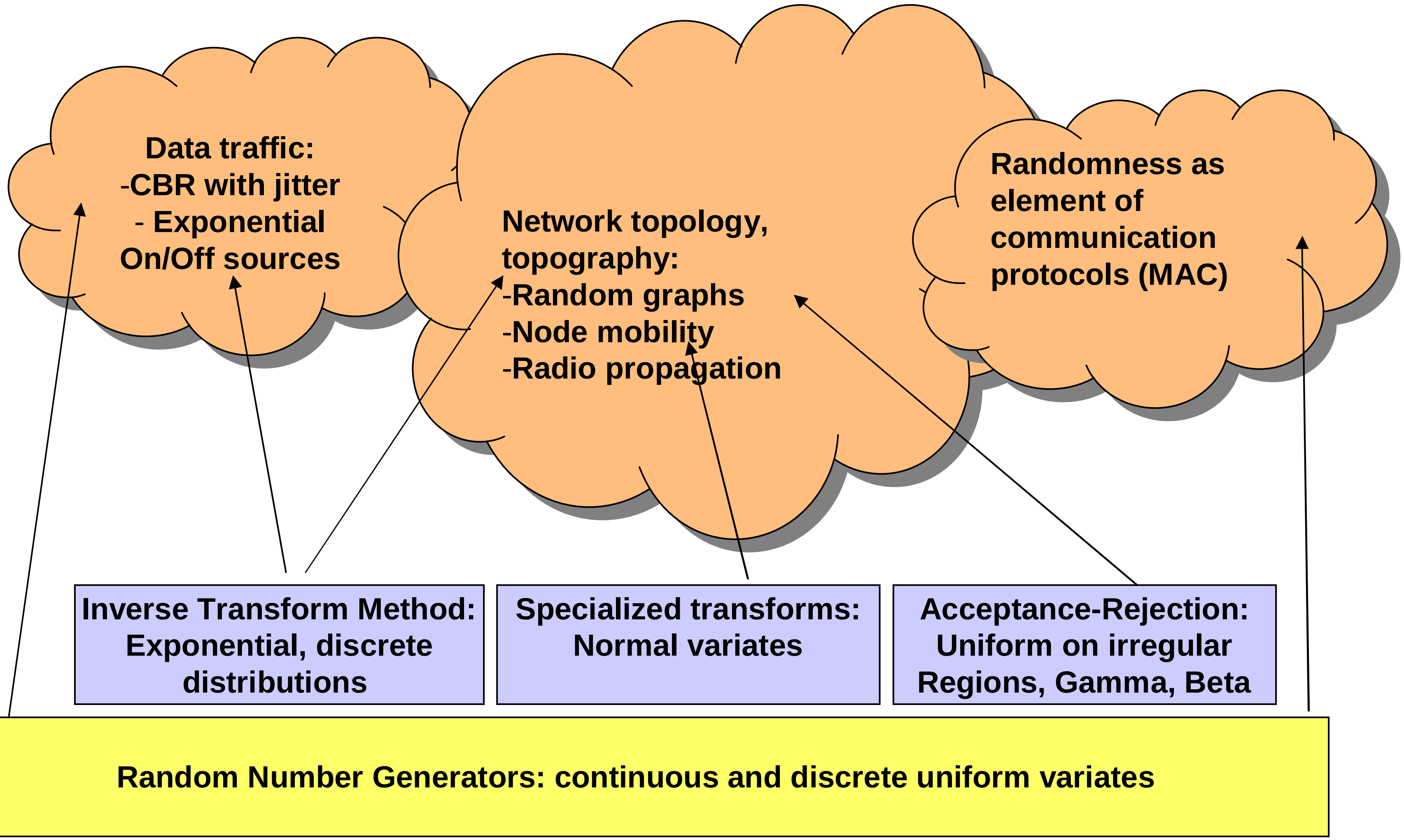
III Acceptance-rejection method: improvement



Wrap-up, summary

- $U(0,1) \rightarrow$ **transform** $\rightarrow$ **desired distribution**
- **Normal variates frequently used to model 'errors' or variation of quantity**
  - **Example: Log-normal shadowing in wireless communications**
- **Acceptance-rejection method**
  - **Less 'direct' than inverse transform method**
  - **Can be used when distribution function does not have closed form expression**
  - **Is used in polar method to generate uniform distribution on unit disc**
- **We know have all major stochastic building blocks for our simulations**

So far … stochastic building blocks and models



References

- **Box-Muller and polar method:**
  - **Press, W. H., Teukolsky, S. A., Vetterling, W. T., Flannery, B. P., *Numerical Recipes in C*, 2nd edition, Cambridge University Press, 1992: chapter 7.**
  - **Knuth, D.E., *The Art of Computer Programming*, vol. 2, 3rd edition, Addison Wesley, 1998: chapter 3.**
  - **Ross, S. M.: *Simulation*, 2nd edition, Academic Press, 1997.**
- **Radio propagation models:**
  - **NS-2 Manual, Dec. 13, 2003, Chapter 18 'Radio Propagation Models'**
  - **Rappaport, T. S., *Wireless Communications – Principles and Practice*, 2nd. ed., Prentice Hall, 2002; Chapter 4.**
- **Acceptance-rejection method:**
  - **Averill M. Law, W. David Kelton: "Simulation Modeling and Analysis", McGraw-Hill, 3rd edition, 2000.**

Event Scheduling

Holger Füßler

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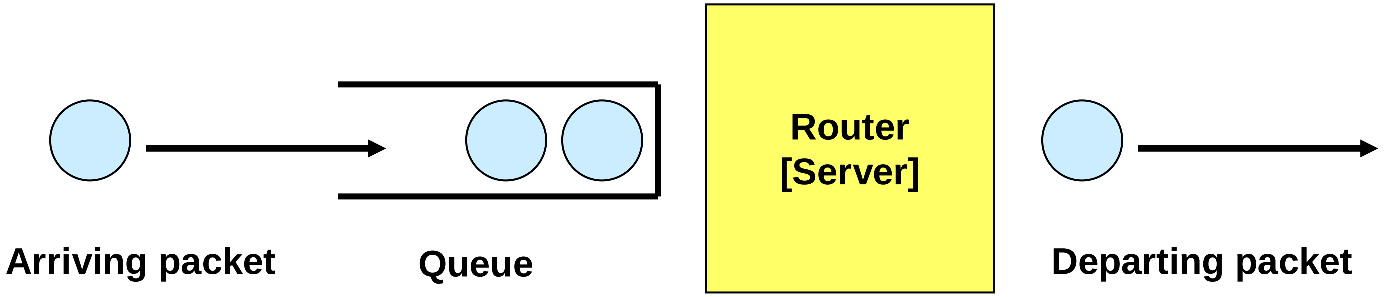
Recap: event scheduling w.r.t. M/M/1 queue



- **Queuing systems as delay models**
- **Arrival process: 'M' for 'memoryless' (thus, exponentially distributed inter-arrival times)**
- **Service process: 'M' for 'memoryless' (thus, exponentially distributed service times)**
- **Number of queuing stations: 1**

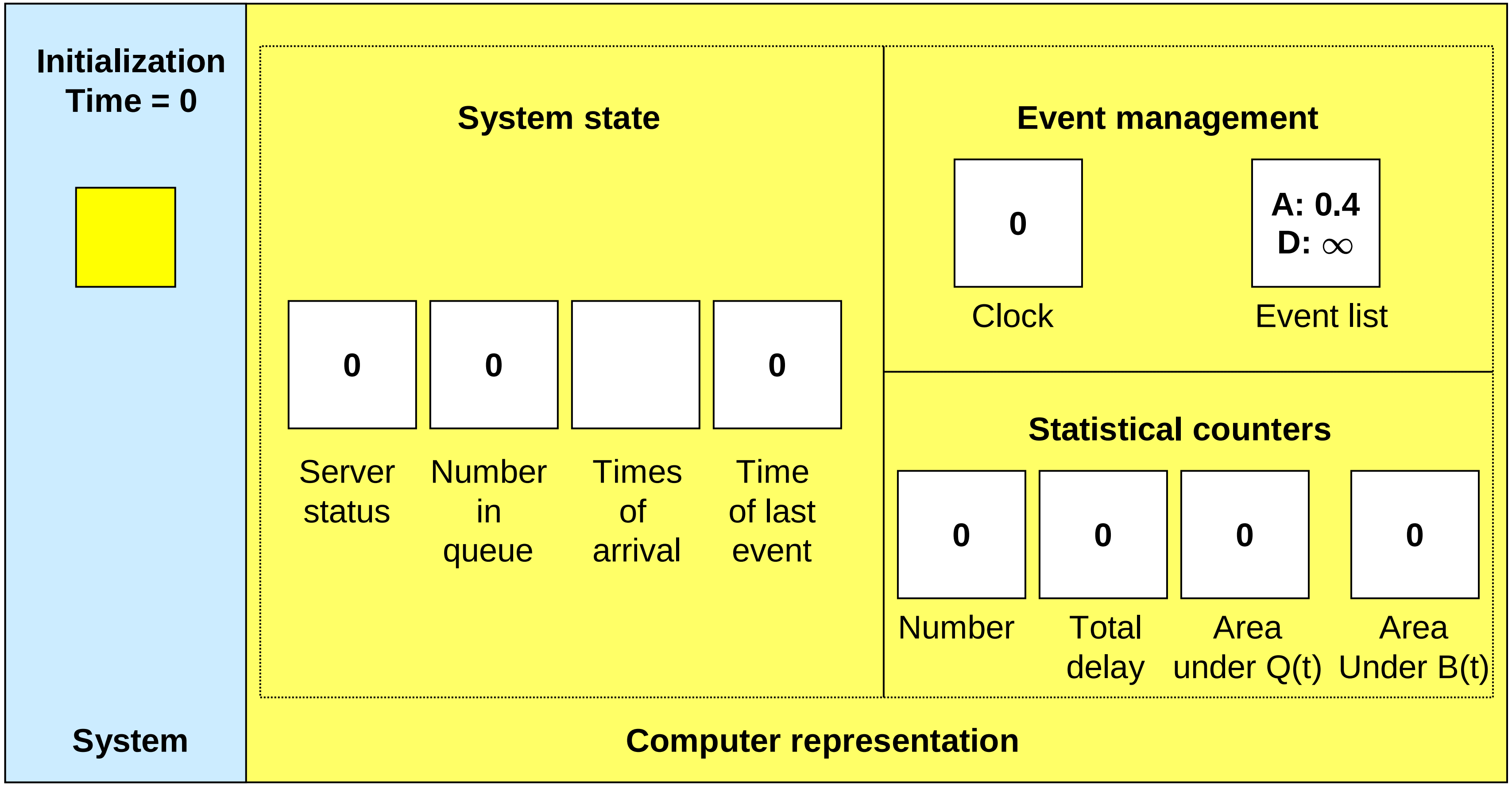
$$f(x) = \frac{1}{\beta} e^{-x/\beta}$$

β=**1.0 s for inter-arrival times**

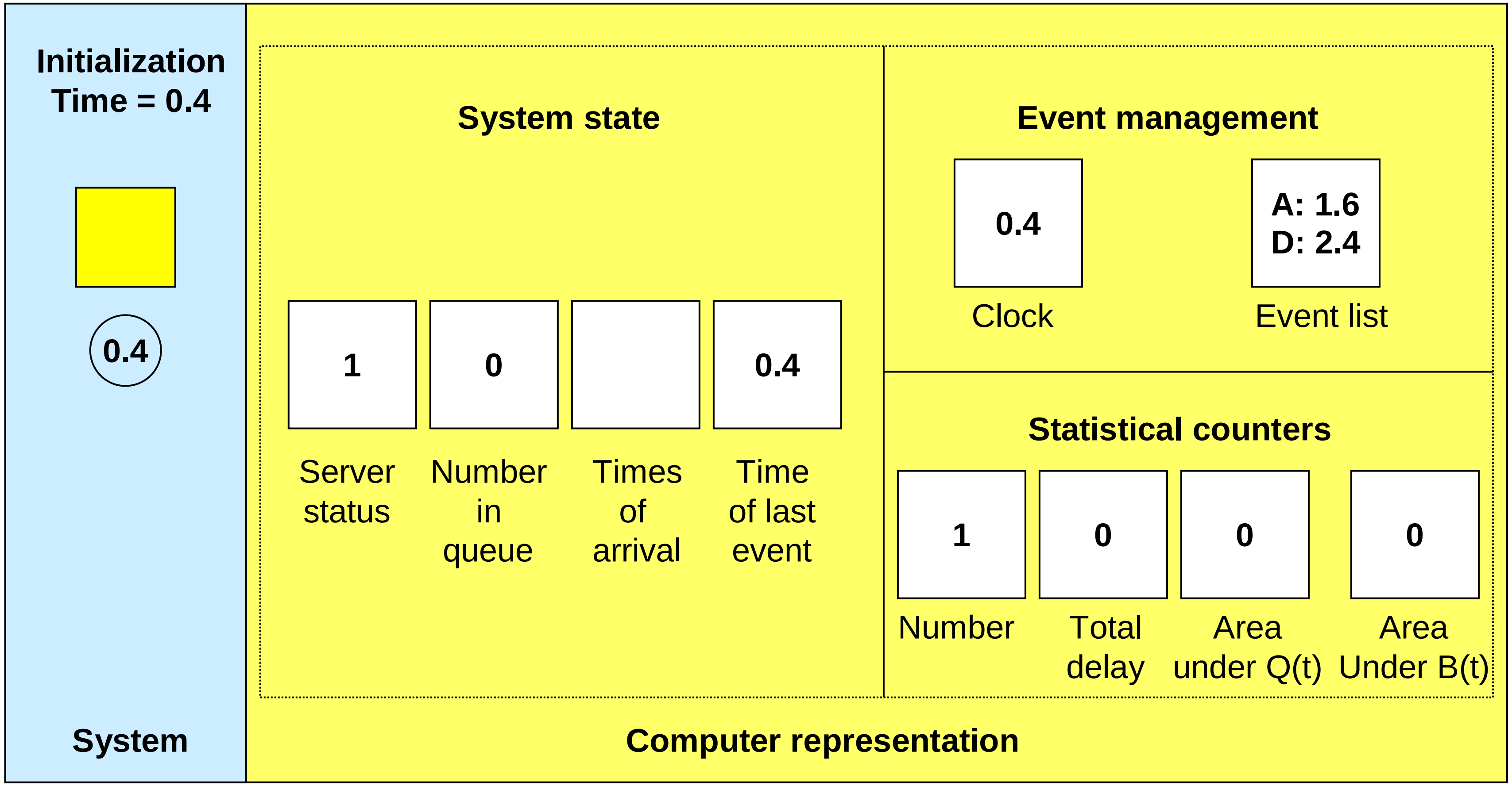
β=**0.5 s for service times**

- **Event: a state transition**
- **Event: depends on system logic and stochastic modeling**

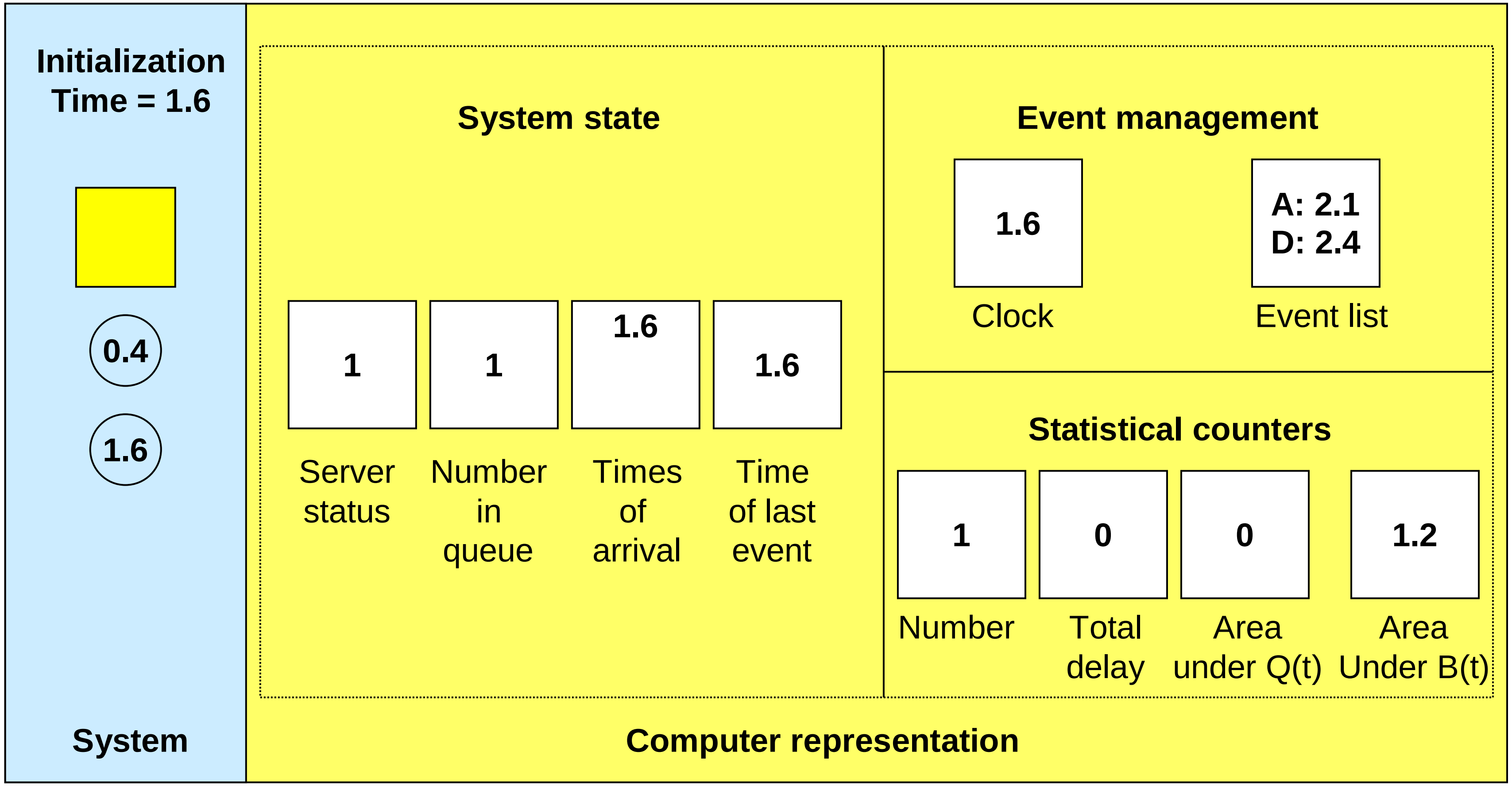
Recap: execute model



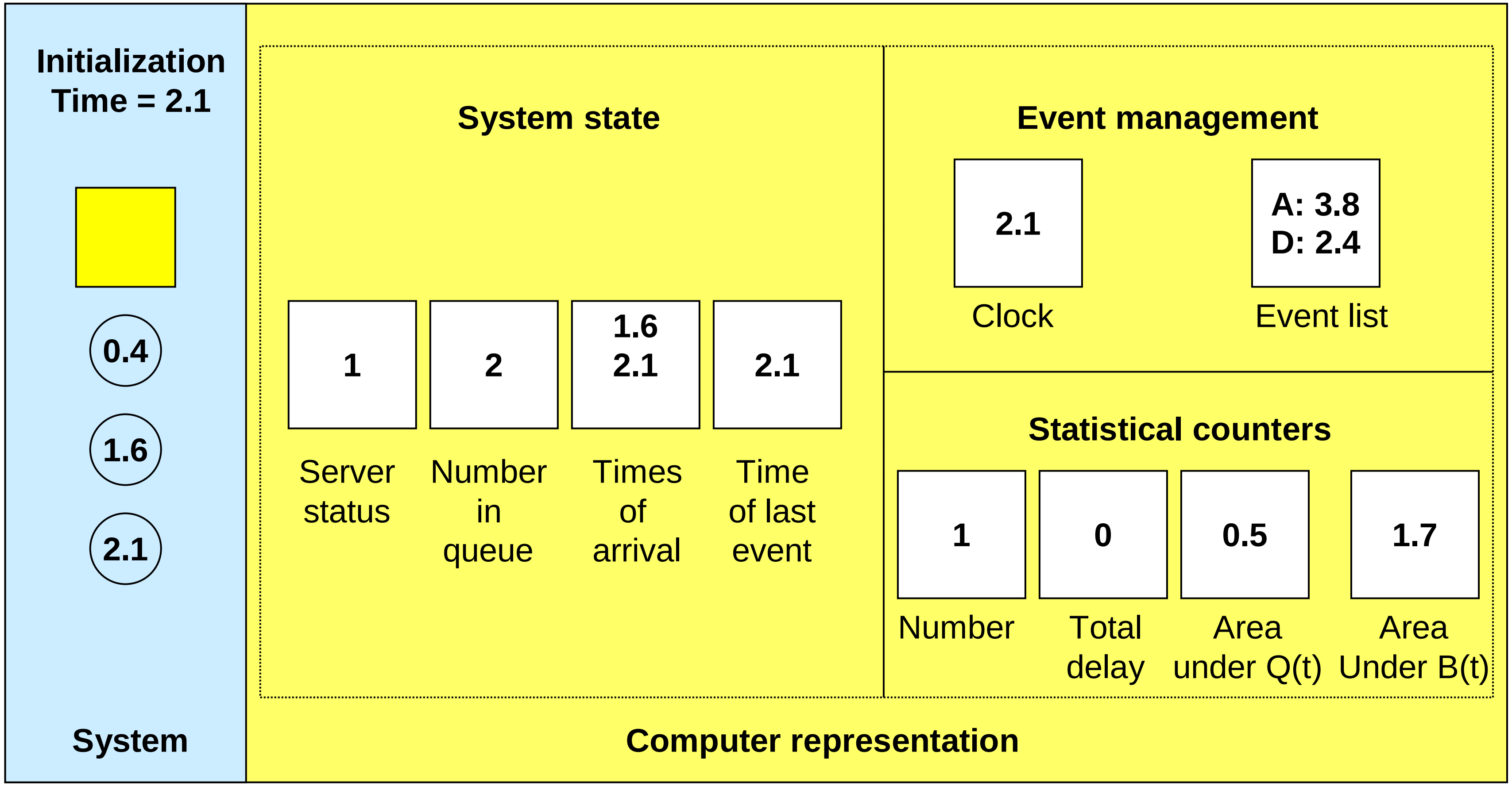
Recap: execute model



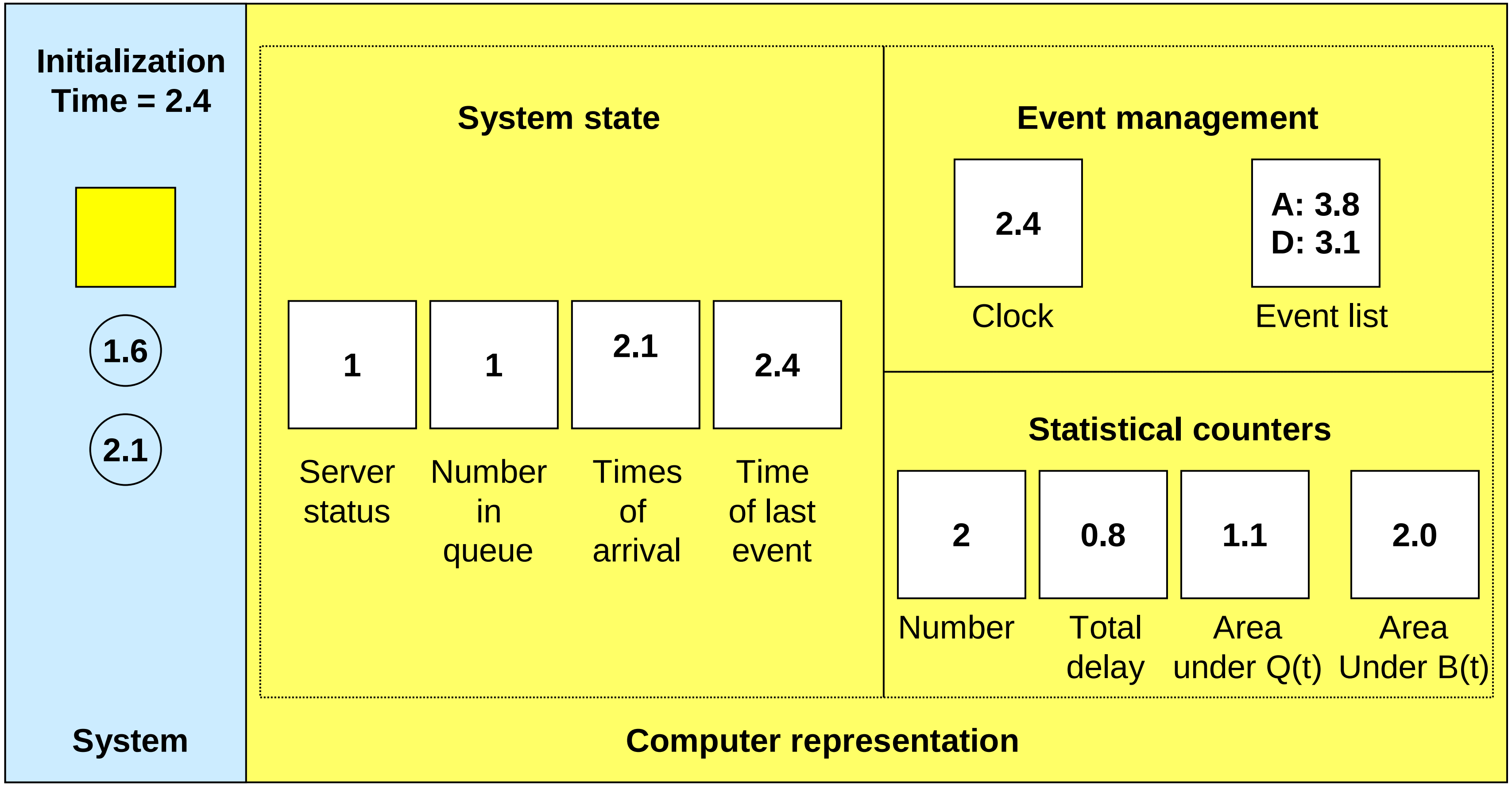
Recap: execute model



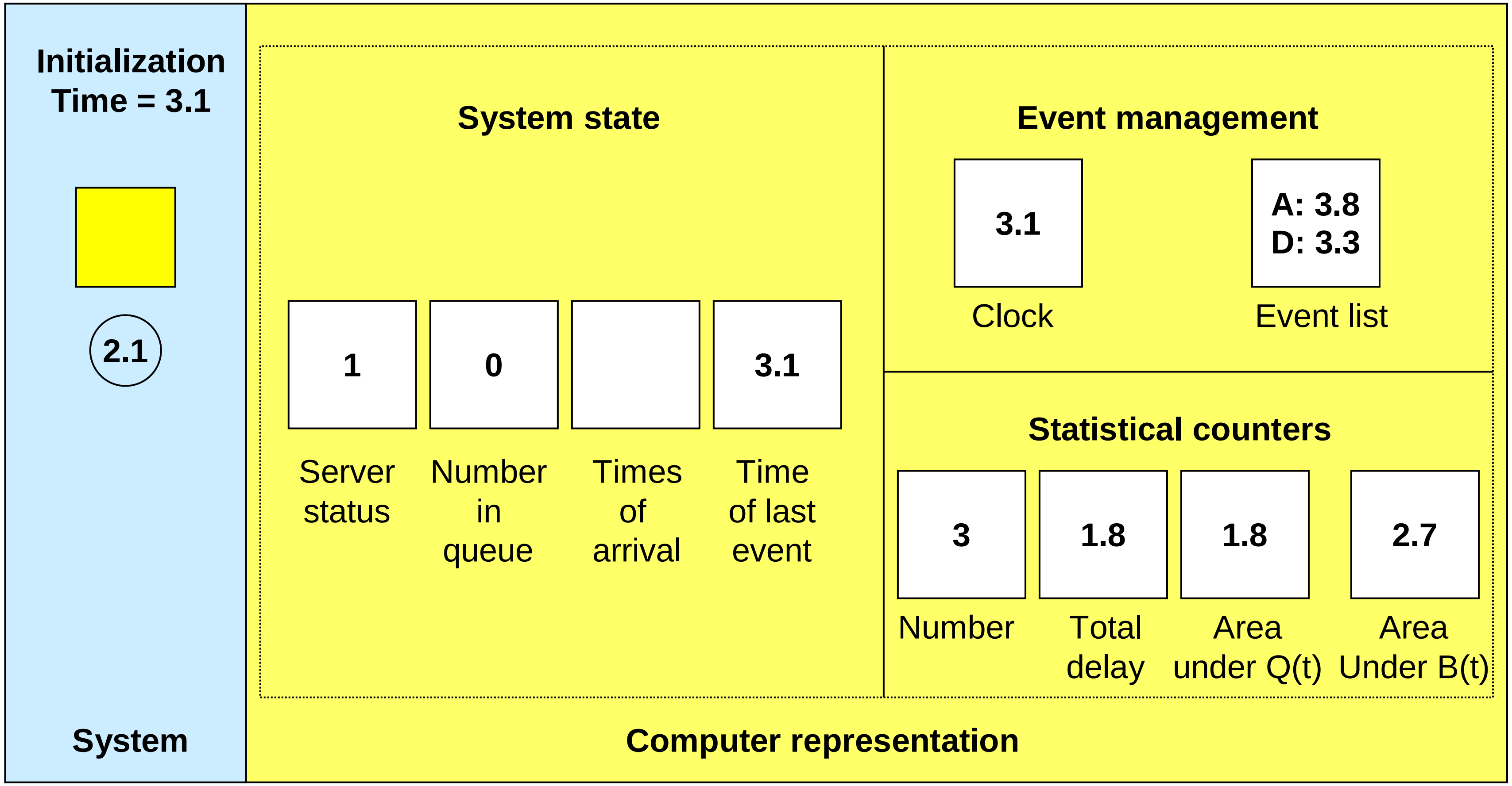
Recap: execute model



Recap: execute model



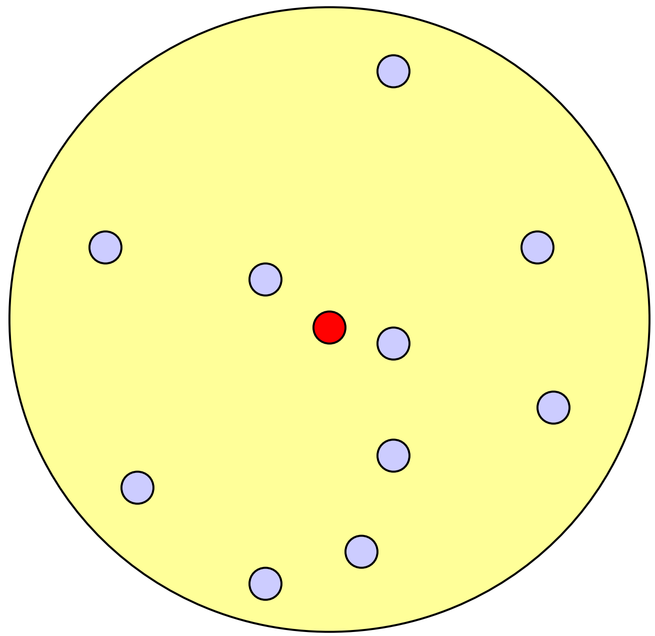
Recap: execute model



… see [LK2000] for continuation of this example

Scalability challenge

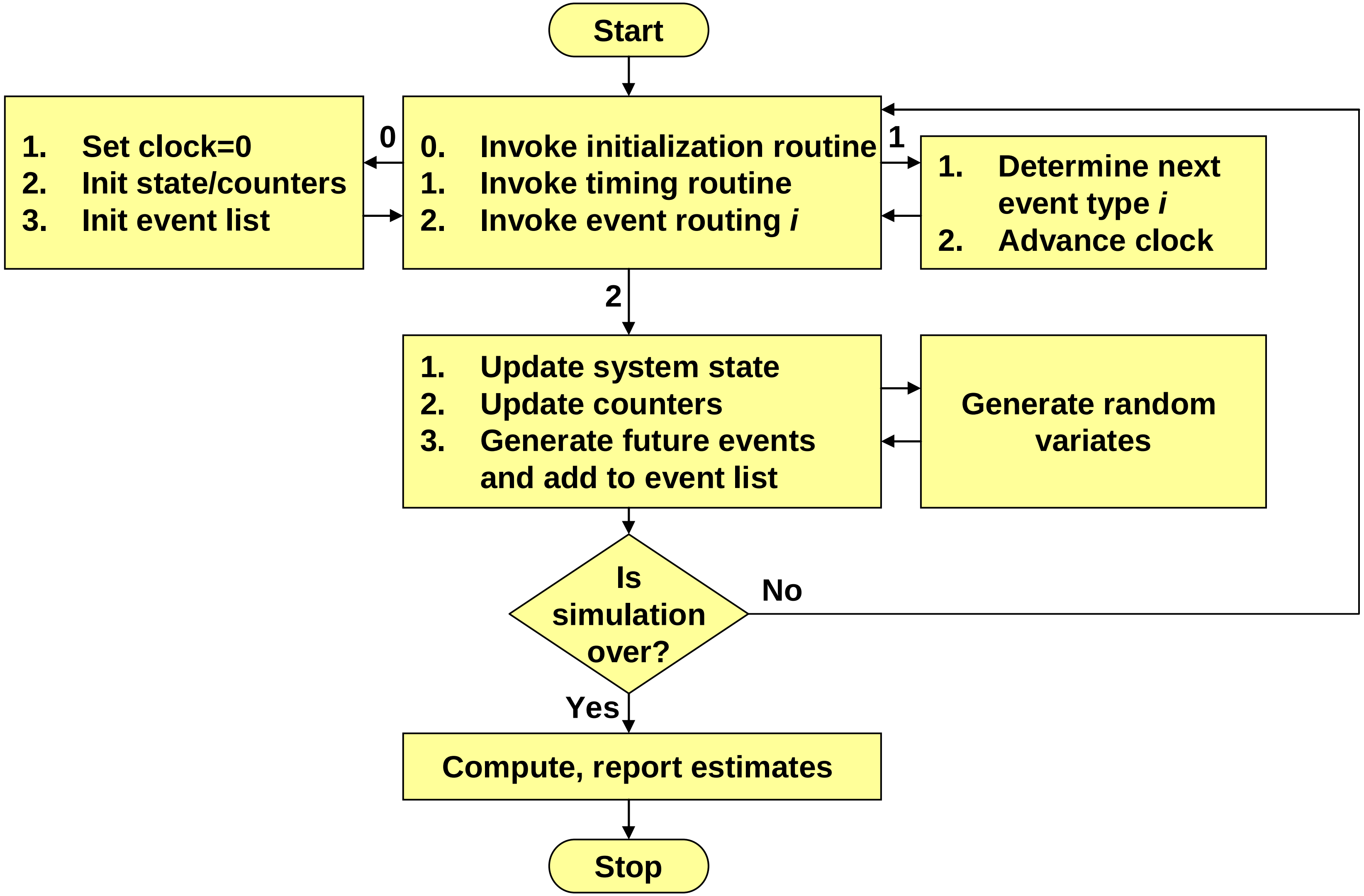
- Example: wireless communication
- For every packet send by a sender, all nodes (at least the ones within transmission range) have to schedule a ‘receive event’.
- Thus, we have at least $O(N^2)$ events where N denotes the number of nodes (sender/receiver).
- This lecture: focus on sequential processing of events



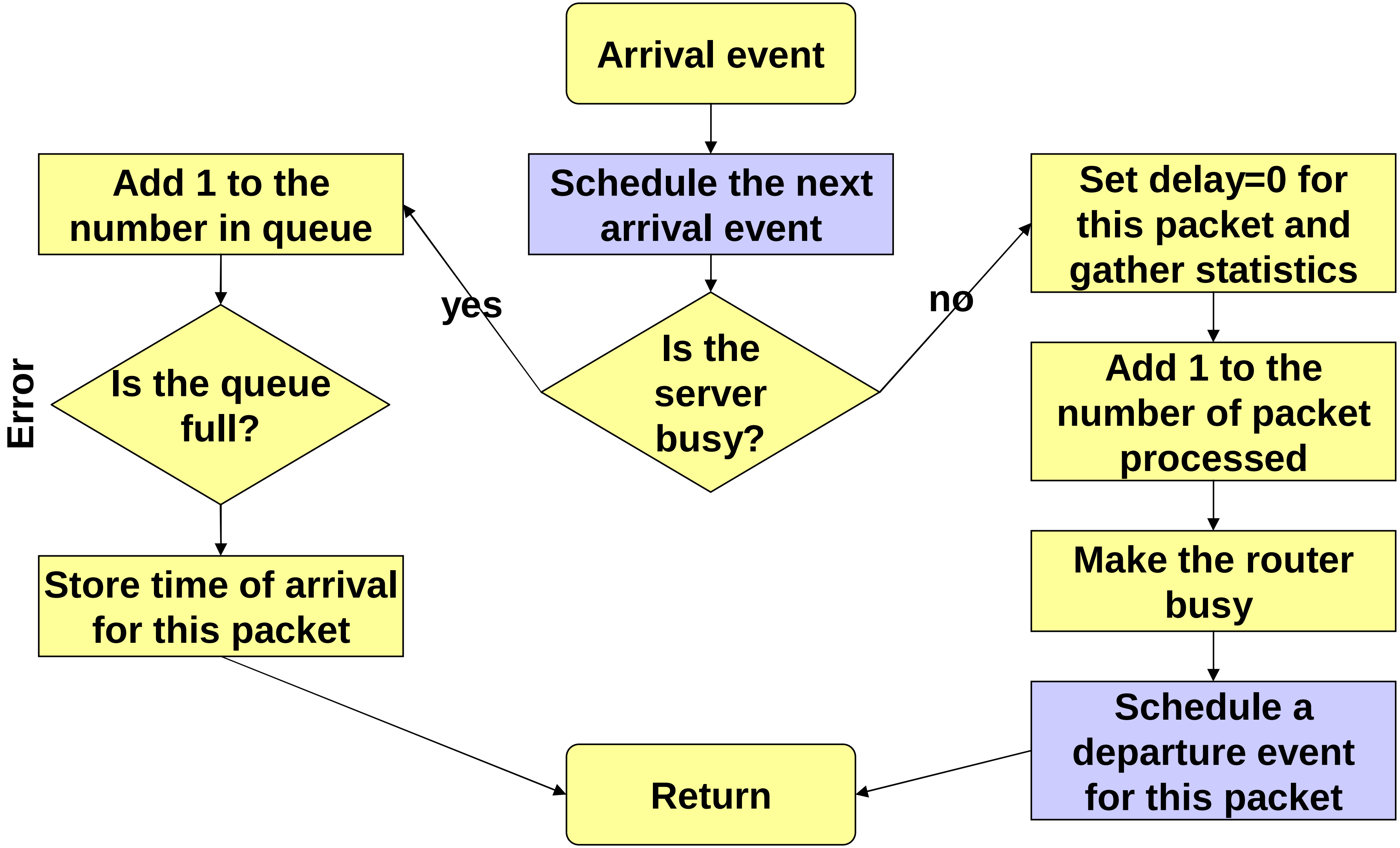
Structure

- » **Part I: Management of discrete events: problem statement**
- » **Part II: Linear lists**
- » **Part III: Heaps**
- » **Part IV: Splay trees**
- » **Part V: Calendar queue**
- » **Part VI: Scheduling in NS-2**

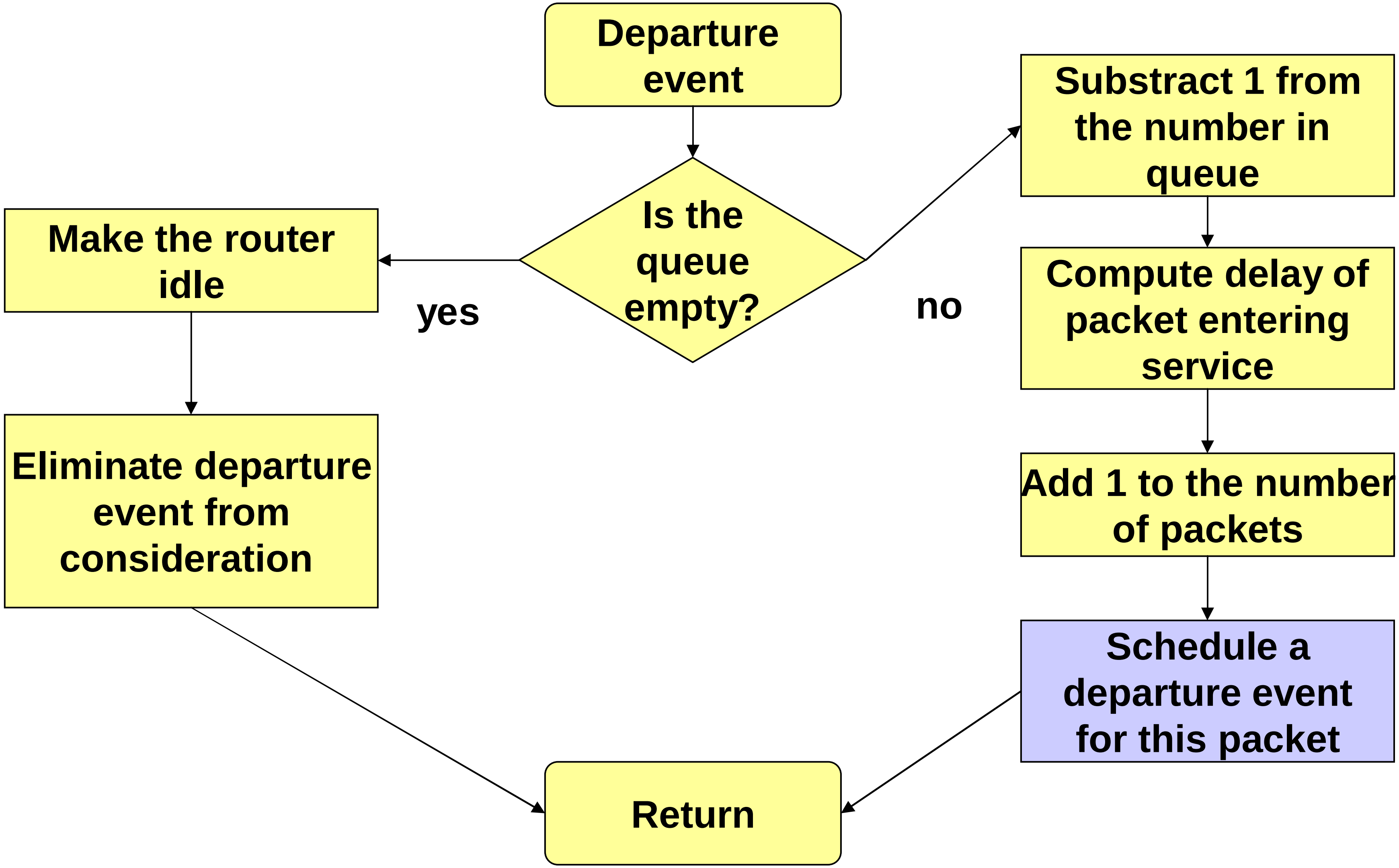
I Discrete event simulation: flow diagram



I Example M/M/1 queue: event type arrival



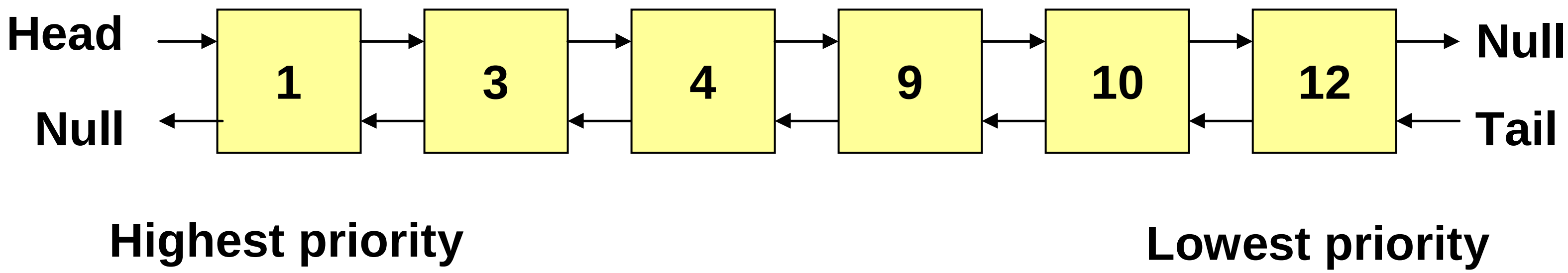
I Example M/M/1 queue: event type departure



I Event management: operations

- Enqueue event
- Dequeue ‘next’ event
- Usually: # enqueued events = # dequeued events
- But: distribution of event types and times can differ drastically
- Example:
  - Enqueue, enqueue, dequeue, enqueue, dequeue, enqueue, dequeue, …
  - Enqueue, enqueue, enqueue, … dequeue, dequeue, dequeue, …
- Required: efficient data structure for event management.
- Priority queues

II Sorted doubly-linked linear list



- Dequeue next event: take first element
  - Costs: O(1)
- Enqueue event according to priority
  - Costs: O(N) where N is the number of events in the list

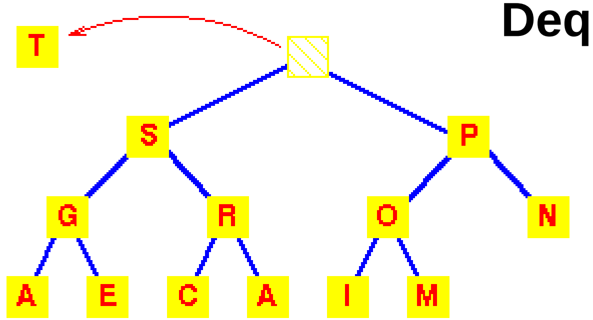
- Knowledge on interval between events can be used to improve insertion process.

III Heaps

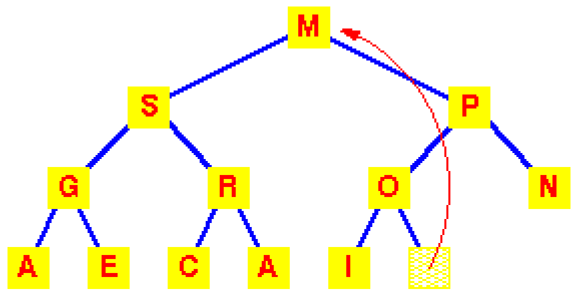
- **Standard priority queue**
- **A binary tree has 'heap property' if**
  - **it is empty or**
  - **any node has a higher priority than its children**
- **Can be easily stored in an array**
  - **Root: A[1]**
  - **Children of A[i] are given by A[2i] (left) and A[2i+1] (right)**
- **Dequeue:**
  - **Remove root**
  - **Put element of current right bound of array to root**
  - **Restore heap property**
  - **Costs: O(log(N))**
- **Enqueue:**
  - **Put new element on current right bound of array**
  - **Restore heap property**
  - **Costs: O(log(N))**

III Heaps: en-/dequeue

Dequeue



1



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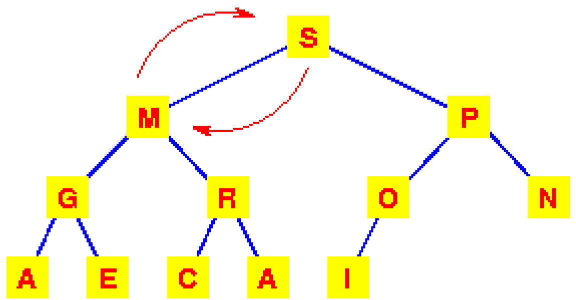

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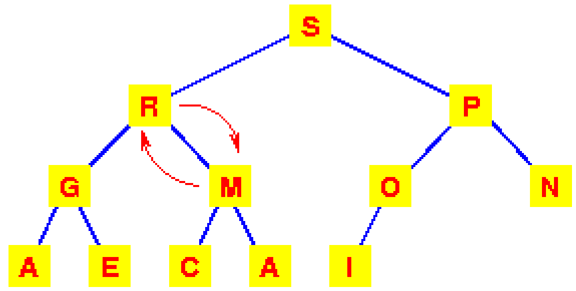
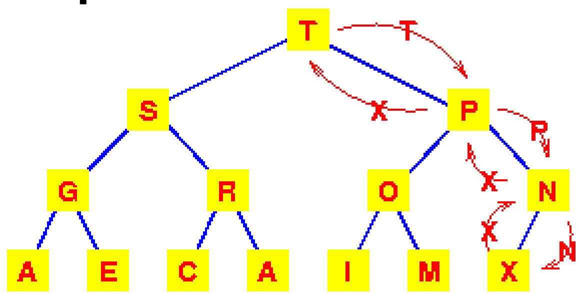


Enqueue

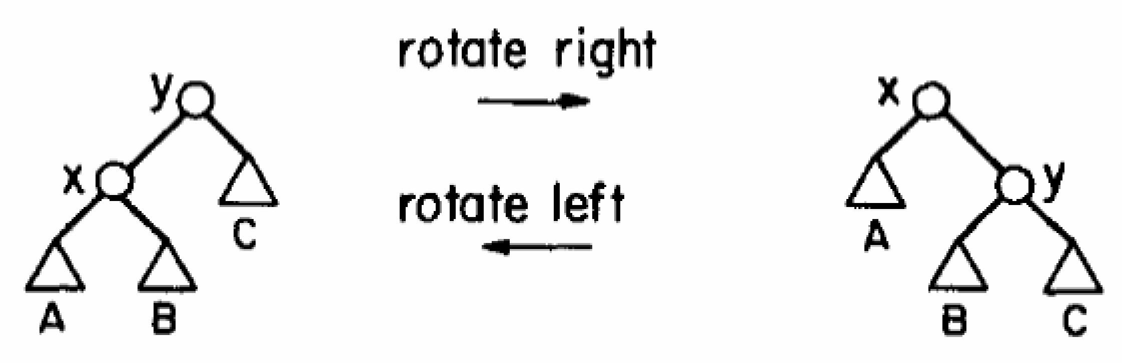


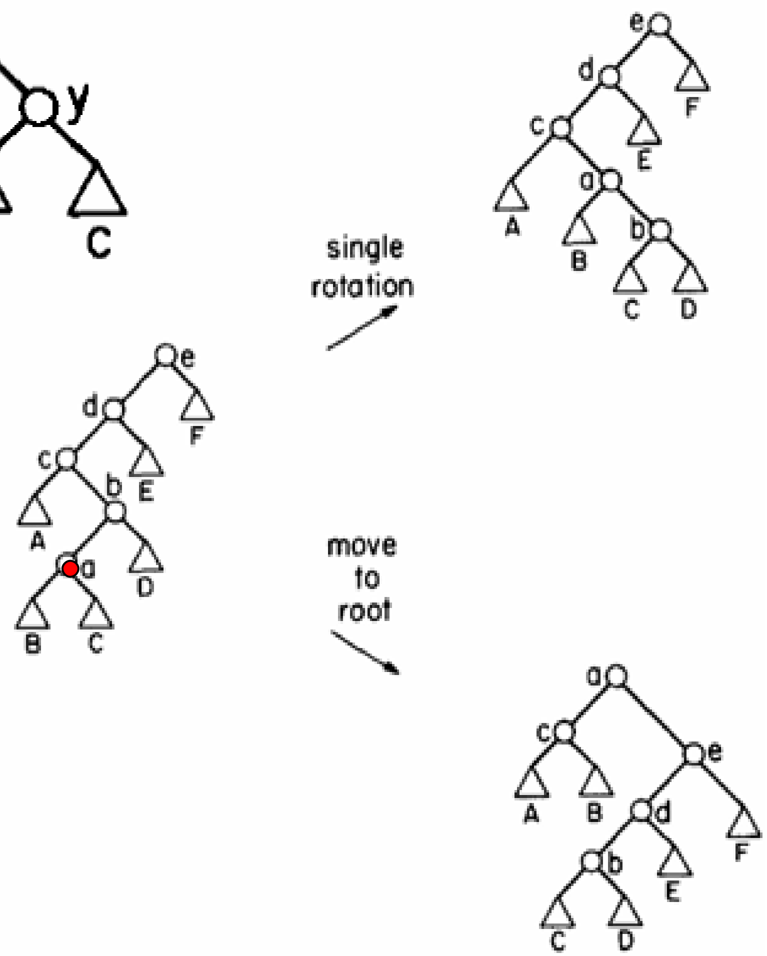
[Source:http://ciips.ee.uwa.edu.au/~morris/Year2/PLDS210/heaps.html]

IV Splay trees

- Splay trees are binary search trees
- Binary search tree: for each node i
  - All nodes in left subtree of node i have smaller priority
  - All nodes in right subtree of node i have higher priority
- Costs for enqueue, dequeue in a sufficiently balanced binary search tree: O(log(N)) where N denotes the number of nodes
- Splay trees: use heuristics to reorg the tree during an en-/dequeue operation. The reorg pays off in subsequent operations.
- Implementation available in NS-2

IV Splaying operations (examples)





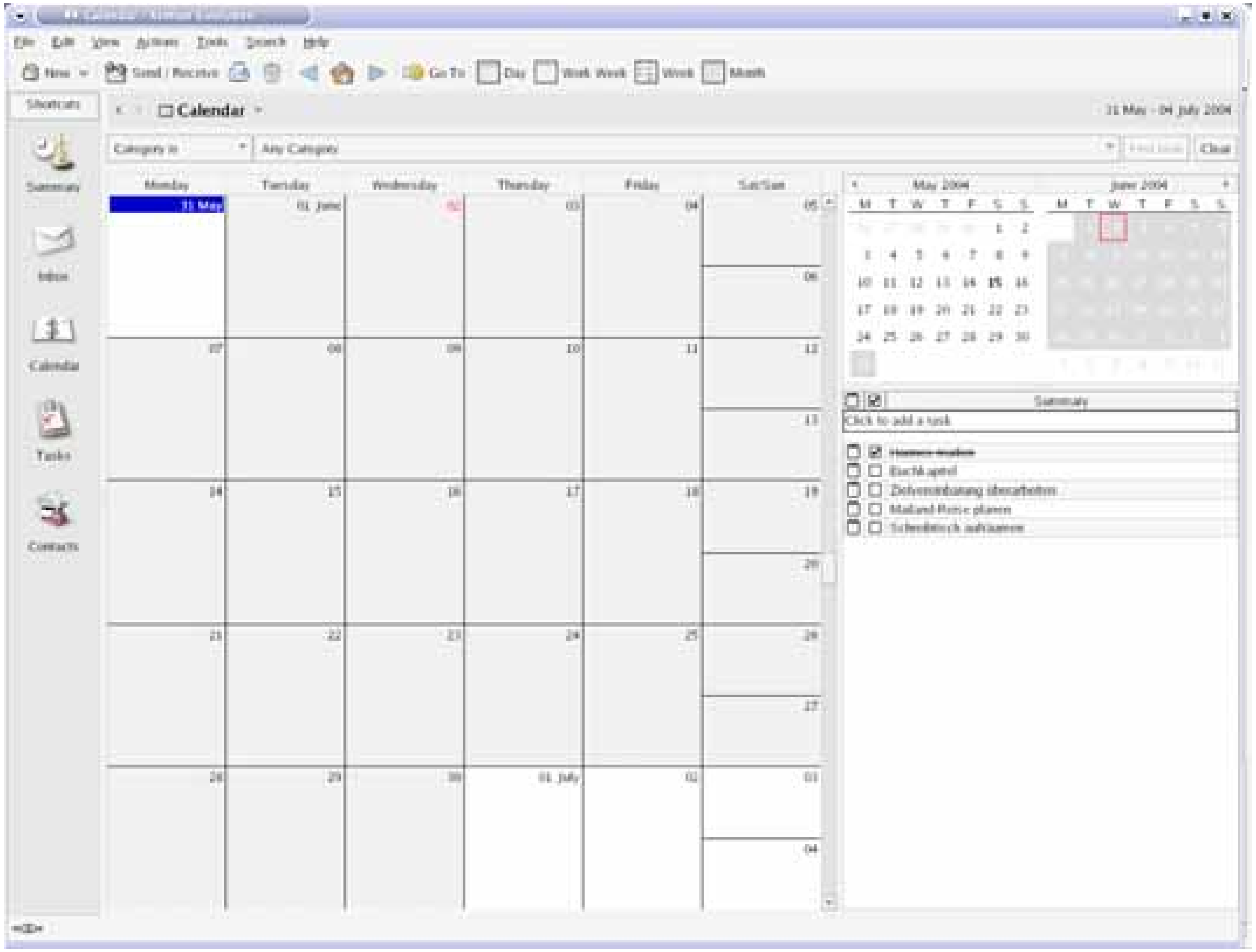
IV Results (Jones, 1986)

TABLE II. Summary of Conclusions

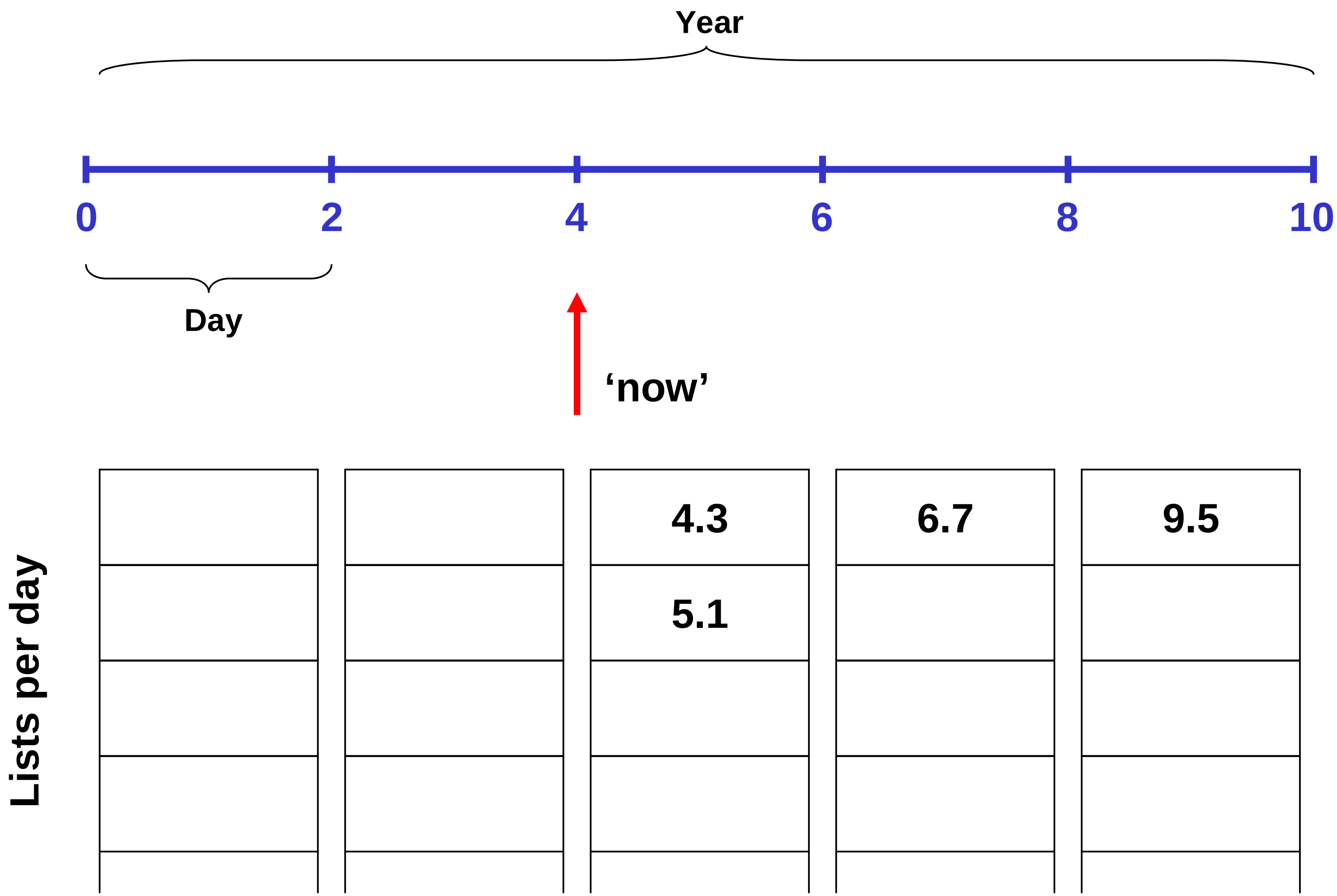
Priority-queue implementation	Code size[a]	Performance		Relative speed[b]	Comments
		Average	Worst		
Linked list	47	$O(n)$	$O(n)$	11	Best for $n < 10$
Implicit heap	72	$O(\log n)$	$O(\log n)$	8	
Leftist tree	79	$O(\log n)$	$O(\log n)$	9–10	
Two list	104	$O(n^{0.5})$	$O(n)$	9–10	Good for $n < 200$
Henriksen's	68	$O(n^{0.5})$	$O(n^{0.5})^c$	1–7	Stable
Binomial queue	188	$O(\log n)$	$O(\log n)$	1–7	
Pagoda	110	$O(\log n)$	$O(n)$	4–8	Delete in $O(\log n)$
Skew heap, top down	56	$O(\log n)$	$O(\log n)^c$	5–7	
Skew heap, bottom up	103	$O(\log n)$	$O(\log n)^c$	4–6	Delete in $O(\log n)$
Splay tree	119	$O(\log n)$	$O(\log n)^c$	1–3	Stable
Pairing heap	84	$O(\log n)$	$O(\log n)^c$	3–6	Promote in $O(1)$

[a] The total lines of Pascal code for initqueue, emptyqueue, enqueue, and dequeue.
[b] 1 is fastest; 11 is slowest. The rankings are based on Figures 12–14.
[c] An amortized bound; single operations may take $O(n)$ time!

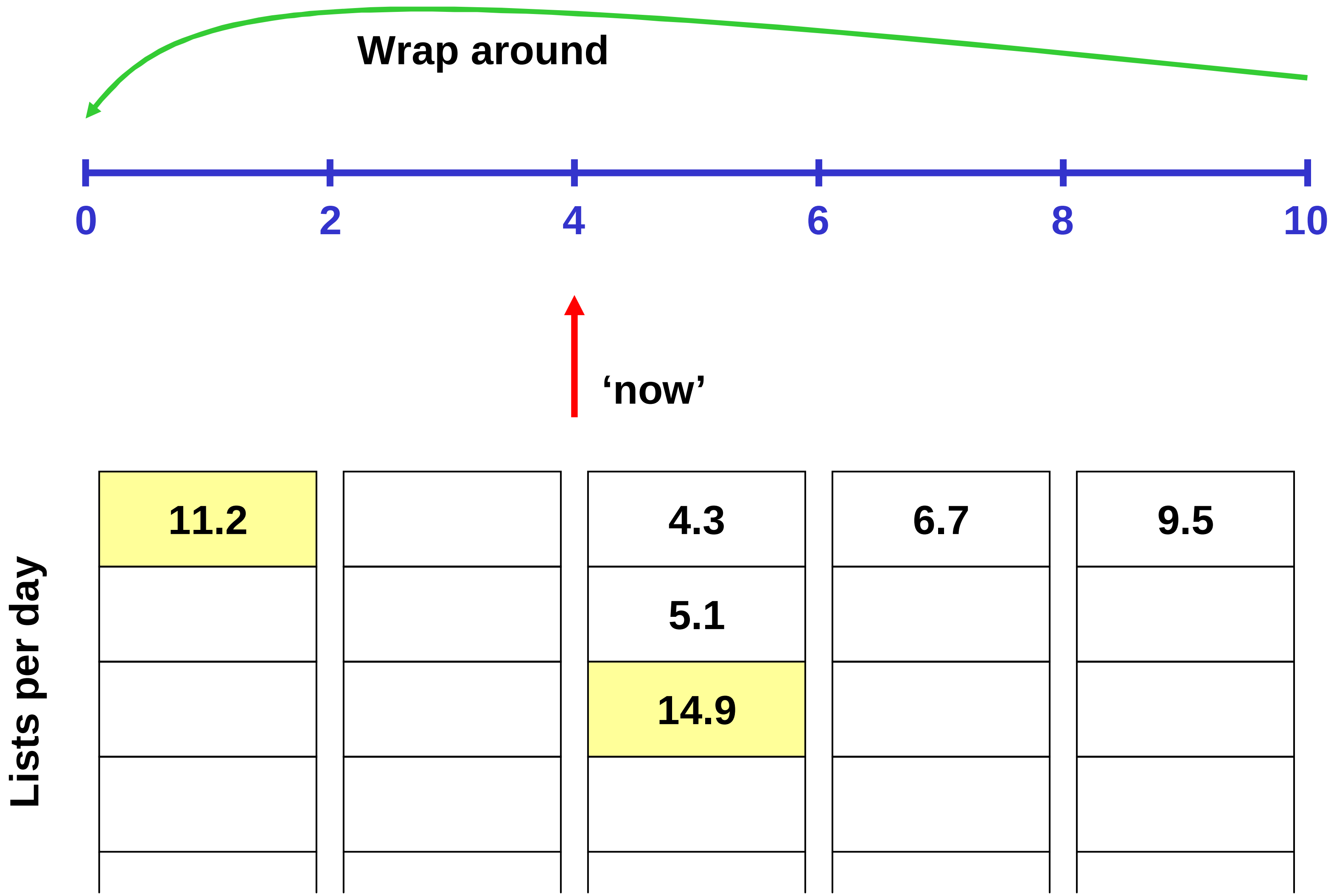
V Calendar queue: basic idea



V Calendar queue: static case I



V Calendar queue: static case II



V Calendar queue: static case III

Operations:

» **Get next event (dequeue):**

- **a) go to next event of current day or go to next day**
- **b) if a) fails for a whole year, use direct search**

Example:

- **500 events between [0,0.1]**
- **500 events between [5.6,5.7]**
- **Length of day: 0.0002**
- **Length of year: 0.1024 (512 days)**
- **Strategy a) needs to cycle through the calendar for 54 year to find the first event of [5.6,5.7]**

» **Insert *key* (enqueue):**

- **Find right day/bucket: *key* % ndays**
- **Insert *key* into ordered list**

V Problems with static case

- **Many keys per day: enqueue operation gets expensive**
  - **Go through list of length O(N) where N denotes the number of events**
- **Only a few keys within many days: dequeue operation gets expensive**
  - **Go through empty days of length O(B) where B denotes the number of buckets**
- **Analysis of problem complexity is not trivial:**
  - **See ‘Optimizing static calendar queues’, K. B. Erickson, R. E. Ladner, A. Lamarca, ACM Tomacs, vol. 10, no. 3, July 2000, pp. 179-214.**

V Calendar queue: dynamic case

Idea: Adjust length of year and days according to current key set.

» **Number of days (buckets):**

- **Whenever the number of keys exceeds twice the number of days, copy calendar to a larger calendar (typically: double size).**
- **Whenever the number of keys is less than half the number of days, copy to a smaller calendar (typically: half size).**

» **Length of day:**

- **Dequeue samples from calendar (typically 25)**
- **Calculate average separation of dequeued events**
- **Set new length of day to average separation (usually multiplied with some factor)**
- **Enqueue samples**

V Evaluation of calendar queues I

- **Assume exponential distribution:**
  - **Mean 1**
  - **Next priority:**
    last priority – ln(rng())
- **Hold operation: dequeue followed by enqueue**

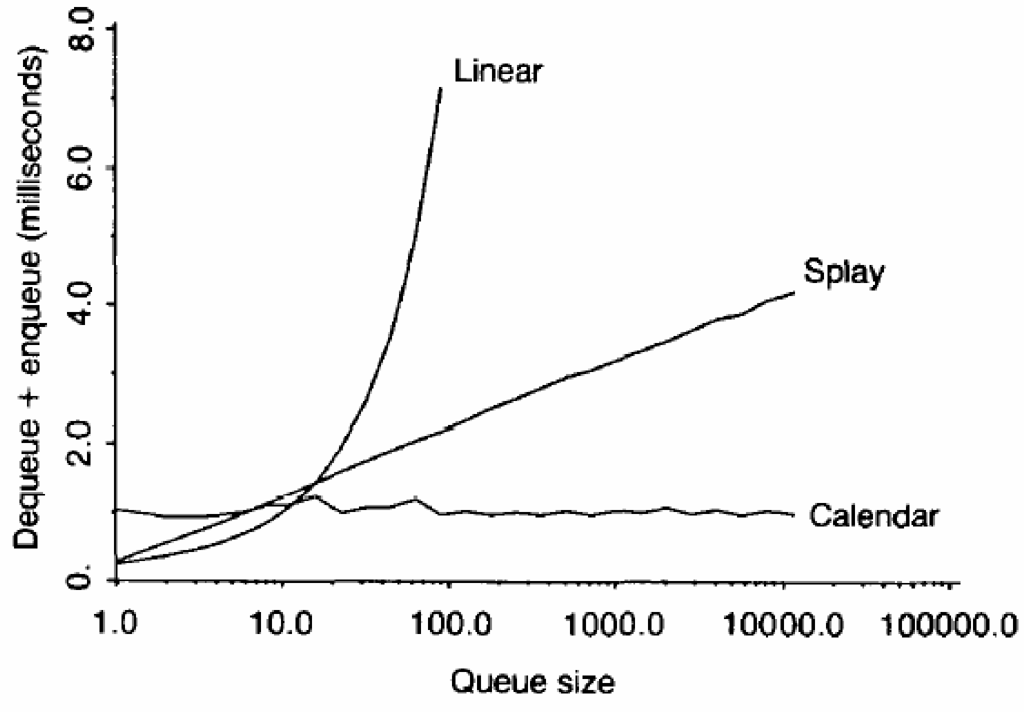


FIGURE 3. Comparison of Hold Times on Texas Instruments PC

[Source: Original paper by Brown, 1988]

VI Scheduling in NS-2

- **Choices are: list, heap, splay tree, calendar queue, real-time**
  - **Real-time: tries to synchronize with real-time clock; experimental**
- **Calendar queue is default**
- **How to deal with events at same time?**
  - If more than one event are scheduled to execute at the same time, their execution is performed on the first scheduled – first dispatched manner.
  - Simultaneous events are not reordered anymore by schedulers (as it was in earlier versions) and all schedulers should yield the same order of dispatching given the same input.
- **Accuracy: see next slide**

VI Precision of the scheduler clock in NS-2

- Precision of the scheduler clock: smallest time-scale of the simulator that can be correctly represented.
- The clock variable for ns is represented by a double. As per the IEEE std for floating numbers, a double, consisting of 64 bits must allocate the following bits between its sign, exponent and mantissa fields.
- sign exponent n mantissa X

  1 bit 11 bits 52 bits
- Any floating number can be represented in the form $X \cdot 2^n$ where X is the mantissa and n is the exponent.
- Thus the precision of time in ns can be defined as $1/(2^{52})$.
- As simulation runs for longer times the number of remaining bits to represent the time educes thus reducing the accuracy. Given 52 bits we can safely say time up to around 2^{40} can be represented with considerable accuracy.

VI Precision of NS-2

- $2^{40} \approx 10^{12}$
- Thus, for a simulated time of 1000 seconds we still have nanosecond accuracy (10^{-9}).

Wrap-up

- Event management requires efficient priority queues: schedule an event, dequeue next event.
- Linear lists, heaps, splay trees, calendar queues
- Efficiency depends on ‘priority increment distribution’
- Calendar queue in general ‘safe guess’
- But: one should check its own priority increment distribution to see whether improvements are feasible
- Precision: with a 64-bit double, there is still a nanosecond accuracy for a simulated time of 1000 seconds.
- Open: is there a paper available with a performance evaluation of priority queues on current Pentium machines?

References

- **William M. McCormack, Robert G. Sagent, *Analysis of future event set algorithms for discrete event simulation*, Communications of the ACM, vol. 24, no. 12, December 1981, pp. 801-812**
- **Douglas W. Jones, *An empirical comparison of priority-queue and event-set implementations*, Communications of the ACM, vol. 29, no. 4, April 1986, pp. 300-311**
- **Randy Brown, *Calendar queues: a fast O(1) priority queue implementation for the simulation event set problem*, Communications of the ACM, vol. 31, no. 10, October 1988, pp. 1220-1227**
- **Daniel D. Sleator, Robert E. Tarjan, *Self-adjusting binary search trees*, Journal of the ACM, vol. 32, no. 3, July 1985m pp. 652-686**
- **NS manual, Dec. 13, 2003.**